NEW ELEMENTS OF *Conick Sections*:

Together with a

METHOD

For their

Deſcription on a *Plane*.

Tranſlated from the French Treatiſe of Mr. De la Hire.

By *Brian Robinſon*.

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To the Reverend

Mr. *John Harris* A.M.

AND

Fellow of the *Royal Society.*

SIR,

SINCE Mathematicks hath been introduced into Physica Speculations, and Men have begun to see thro' the empty Jargon of the Schools, to distinguish betwixt Chymera's and Solid Knowledge, Learning has received vast Improvments: And I may

venture to say, that you by your late excellent *Work* with which you have obliged the World, have done more Service to true Philosophy, and useful Learning, than any one single Person besides; for those many and great Advances which have been made within this last Century, in particular parts of Learning, you have with great Improvments and good advantage brought into one view, and under the modest title of *Lexicon Technicum*, have given us a compleat body of Science. Nor

Nor is your Goodneſs leſs conſpicuous then your other Qualifications, your generous Converſation and diſintereſted ſincere Friendſhip, makes you eſteemed and beloved by all thoſe who have had the happineſs of your more intimate acquaintance; and I am ſure I may with better reaſon ſay of you, what that great General *Alexander* did of his Maſter *Ariſtotle*: *that my Being indeed is a Debt due to my Parents, but my well Being is entirely owing to you.*

DEDICATION.

Be pleaſed, Sir, to accept with your wonted goodneſs, this ſmall preſent which I here make you, and look upon it as a mark of that juſt Eſteem and Reguard I have for my beſt Friend. I own it is yours by Right, for 'twas you that Engaged me in it, and have ſince Reviſed the Sheets as they came from the Preſs.

I need not ſay any thing of the Author I have here Tranſlated; for your putting me upon it ſufficiently ſhows the value of the Original,

and

and your Correcting the Sheets, in my Abſence, will, I hope, in ſome meaſure recommend the Performance.

May you Live long and happily, for the ſervice of the Church and your Country, the Encouragment of good Letters, and the Ornament of the Mathematicks; which is wiſhed by none more earneſtly than by,

SIR,

Your moſt Obedient
and Humble Servant,

B. ROBINSON.

The *Author's* PREFACE.

SOME Years ago I Publiſhed a Treatiſe of Conick Sections in a new Method, and Demonſtrated their Principal Properties from the Cone: But ſuch as had not been ſufficiently enured to Demonſtrations about the Interſections of Planes and Solids, found it difficult to underſtand them, tho' they were very Simple and Plain when once comprehended. This put me upon ſeeking out another Method, whereby, without making uſe of the Cone,

Cone, and by only drawing Curve Lines upon a Plane, I might demonſtrate the ſame Properties of the Conick Sections: And after having tryed that Method as being the moſt Simple and eaſie of all others, I laid it aſide at laſt, not being able to Surmount at that time all the difficulties which occurred. But I was ſatisfied with having reduced the Conick Sections to a Plain, which I called Plane Conicks; *and I applied to thoſe* Plain Sections *the ſame Demonſtrations which I had before made for the* Solid. *And I can ſay, that That Work had the good fortune to have the Approbation of many Learned Geometers.*

But altho' it be very Advantagious to pleaſe the Learned, *yet we ought by*

by no means to make that the Sole Object and Ultimate End of our Studies, and to neglect the Inſtruction of thoſe that are deſirous to underſtand things of this Nature; and I think they ought to be contented when they are ſhewed ſeveral ways of doing the ſame thing; for then every one may make choice of that which he likes beſt.

Without ſtaying to give you an Enumeration of ſuch Authors as have Written well on this Science, or ſhewing you the different Methods of Deſcribing theſe Curves uſed by them; I ſhall only inſiſt on the difference that there is between my Method, and that of Mr. de. Witt, *which hathbeen juſtly thought the beſt.*

He makes uſe of a particular Method

thod for the Deſcription of each Section, and which, eſpecially in the Parabola and Hyperbola, is embaraſſed with many Lines, interſecting one onother in certain Angles; by which means 'tis difficult at firſt for the Learner to get a clear and plain Idea of the Formation of theſe Curves; whereas the Deſcription that I uſe, and which is taken from the Principal Properties of the Foci, hath nothing for its Rule but one ſingle Line, which in the Ellipſis is equal to the Sum, in the Hyperbola, is equal to the difference of two other Lines drawn from the Foci to a point in the Curve deſcribed: And in the Parabola, the ſum and difference are found together; for beſides its Focus which is determined in the

Axis

Axis, *if you ſuppoſe alſo another in the* Axis *at an Infinite Diſtance, within the* Parabola *below, 'tis evident that the ſum of theſe Lines which ſhall be drawn from any point in the Curve of the* Parabola, *to thoſe two* Foci, *ſhall be equal to one and the ſame Line drawn from the Infinitely Diſtant* Focus *to a Line, which being perpendicular to the* Axis, *ſhall meet it in a point which is as far diſtant from the* ~~*concourſe*~~ *of the* Parabola *as the* Focus ~~*hath determined.*~~

And if you ſuppoſe another Focus *Infinitely Diſtant in the* Axis *above, without the* Parabola, *'tis alſo evident, that the Difference of two Lines drawn from any Point in the Curve of the* Parabola, *to that Indetermined* Focus

cus, *and to the other determined one ſhall be equal always to one and the ſame Line.*

Theſe are the Properties, and theſe are of very great uſe in Geometry, eſpecially in Catoptricks, Dioptricks *and* Aſtronomy.

I Believe Mr. de-Witt *would not have uſed ſo compounded a Method for the Deſcription of theſe Curves, had he not thought that the Demonſtrations would have been more eaſie and Simple on that account: Which he could not bring to paſs in the Ellipſis, when he endeavoured to Demonſtrate the Properties of the Diameters. The Demonſtration which I have given you, hath ſome Conformity with that of the Antients, but 'tis much more ſimple and Plain.* This

The Author's

This Book contains all the Elementary Properties of the Conick Sections, and there is no more Geometry required, than to underſtand thro'ly the firſt Six Books of Euclide's *Elements; or the Subſtance of what is contained in them, as it hath been demonſtrated divers ways by many Learned Geometers. I have by no means changed the Names which* Apollonius *gave to the Curves, both becauſe cuſtom hath familiarized them to us, and alſo becauſe I have demonſtrated in their order, the Properties on which they depend. The* Parabola *takes its Name from the Equality, that there is between the Squares of the Ordinates to the Diameter, and the Rectangles under the Parameter and the* Abſciſſæ: *As I have*

have demonſtrated in the 1. *Corollarie of the firſt Propoſition, and in the* 14 *of the* Parabola.

The Ellipſis *is ſo called becauſe the Rectangles under the Parameter and the* Abſciſſa *or the Parts of the Diameter intercepted between the Vertex and the ſeveral Ordinates, are leſſer than the Squares of ſuch Ordinates, by a Rectangle ſimilar to thoſe Rectangles. As I have ſhewed in the* 5 *and* 18 *Propoſitions about the Ellipſis. In the Hyperbola the aforeſaid Rectangle under the Parameter and the* Abſciſſa *exceeds the Squares of the ſeveral Ordinates, by a ſimilar Rectangle, &c. as I have demonſtrated in the* 3 *and* 22 *Propoſitions of the Hyperbola.*

The Author's

You will find in this Treatiſe, the moſt uſeful Part of the Firſt Book of Apollonius; *The Properties of the Aſſymptotes, which he delivers in his Second Book, and the firſt Propoſitions of the third, and thoſe about the* Foci: *Which are the moſt conſiderable.*

As to the Proportions which I make uſe of, I cite only Compoſition, Diviſion *and Equality of* Ratio's; *becauſe I ſuppoſe my Reader knows how to change* Alternately, *or to* Invert *or* Convert *the Terms of* 4 *Proportionals*, &c.

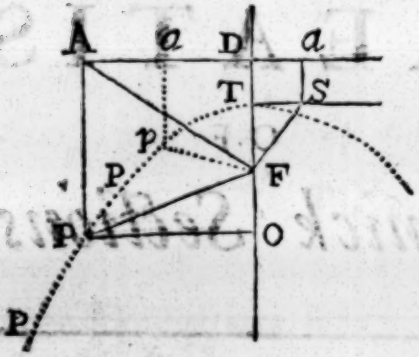
A TREATISE OF *Conick Sections.*

PARABOLA.

The Generation of the PARABOLA.

If a right Line as AD *be drawn upon a Plain, and a Point as* F *be taken without this Line; I say, an infinite Number of Points as* P, P, P, *may be found in such an Order, that the Line* FP *drawn from the Point* F *to*

each Point P, *ſhall be equal to* P A, *drawn from the ſame Point* P *perpendicular to* A D.



FROM any point at pleaſure in the Line A D (ſuppoſe A) having drawn A P perpendicular to A D, and likewiſe the Line F A; make the Angle A F P equal to the Angle F A P, and the point P will be one of the points requir'd: and after the ſame manner an infinite Number of others may be found.

COROLLARY.

Thro' the point F having drawn the Line F D perpendicular to A D; it is evident that

P P (the Line which passes through the points P, P,) bisects F D in T, and that the Line P P T increaseth infinitely, and likewise runs off infinitely from the Line F D produc'd down towards the part F.

DEFINITION I.

The Curve Line P, P, T, form'd by being drawn through the points P, P, is call'd a *Parabola.*

DEFIN. II.

The point T is call'd the *Vertex*, and the point F the *Focus* of that *Parabola.*

DEFIN. III.

The Line D F O the *Axis, and any part of it as* O T, *lying between the Vertex and any point, as* O, *is call'd the* Abscissa.

DEFIN. IV.

The Line P O drawn from any of the points P of the *Parabola*, perpendicular to the Axis, is call'd an *Ordinate* to the *Axis.*

DEFIN. V.

All the Lines within the *Parabola*, which are drawn parallel to the *Axis*, are call'd *Diameters*.

DEFIN. VI.

A right Line which meets the *Parabola* only in one point, and which does not cut it, is call'd a *Tangent* to it in that point.

PROP. I.

The preceding Generation of the Parabola *being ſuppos'd: I ſay, the Square of any Ordinate as,* PO, *is equal to a Rectangle under twice the Line* FD *multiply'd into* TO *that part of the Axis which is contain'd between the Vertex* T, *and* O *the point where it meets the Ordinate.*

Let TD $= a$, conſequently FD will be $= 2a$; and let FO $= b$: DO $=$ AP $=$ FP *by the Generation of the Parabola*, and DOq $=$ FPq, but FPq $=$ POq $+ bb$, and DOq $= 4aa + 4ab + bb$; therefore

fore $PO^q + bb = 4aa + 4ab + bb$, and taking away bb from both ſides of the Equation, there remains $PO^q = 4aa + 4ab$, but $4aa + 4ab = 2FD \times OT$, therefore $PO^q = 2FD \times OT$. W. W. D.

COROL. I.

It is evident from the foregoing *Prop.* that the Square of each Ordinate is equal to a Rectangle, under a Line, which is the double of FD, *and conſequently an invariable Quantity*, and TO the *Abſciſſa*, or that part of the *Axis* intercepted between the *Vertex* T and O the Foot of the Ordinate.

DEFINITION.

The Line equal to 2 FD is call'd the *Parameter* of the *Axis*, and by ſome the *Latus Rectum*.

COROL. II.

It is evident that the *Focus* is Diſtant from the extremity of the *Axis*, or *Vertex* $\frac{1}{4}$ part of the *Parameter*.

COROL. III.

It is likewiſe evident, that the Squares of the Ordinates are to one another, as their

reſpective *Abſciſſæ*, or the parts of the *Axis*, comprehended between the *Vertex*, and thoſe points where each of 'em meets its reſpective Ordinate, for all of them are equal to Rectangles, whoſe Baſes are thoſe *Abſciſſæ*, and whoſe common Altitude is the Parameter; *but Rectangles of the ſame or equal Altitudes are as their Baſes.*

PROP. II.

The right Line T S, *drawn through the point* T, *parallel to* P O, *touches the* Parabola *in the ſame point* T.

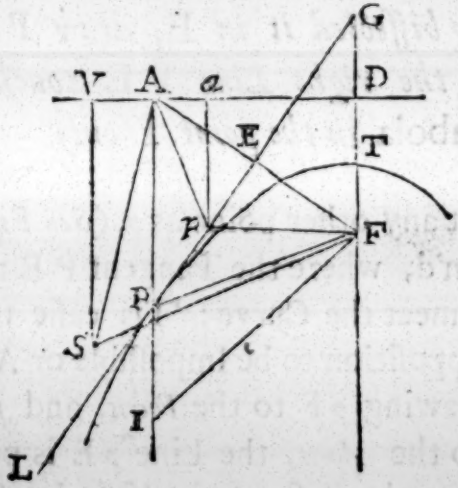
1*ſt*. If it be poſſible let the Line T S (*See Fig. in pag.* 2.) meet the *Parabola* in any other point as S; then having drawn S *a* parallel to the *Axis*, F S = S *a* *by the Generation*; wherefore F S = F T, (becauſe F T, T D, and S *a* are equal) which is Abſurd: For in the Right Angled Triangle F T S, the ſide F S which ſubtends the Right Angle is greater than either of the other ſides.

2*ly*. It

2*ly*. It is ſtill leſs poſſible that T S ſhould ever fall within the *Parabola*, for then the *Parabola* would fall between D*a* and T S, which is impoſſible from the Generation: Therefore the Propoſition is true.

PROP. III.

The ſame things being ſuppos'd as before; I ſay, that a Diameter, as P I *can meet the* Parabola *only in one point* P.



Suppoſe

Suppoſe the Diameter meet the *Parabola* in another point as I, the Line I F will be = I A *from the Generation*, but P F = P A for the ſame Reaſon; wherefore P F + P I will be = I F, which is Abſurd; for any two ſides of a Triangle taken together are greater than the third, wherefore the Propoſition is true.

PROP. IV.

The ſame things being ſtill ſuppos'd as above, draw the Line F A, *and having biſſected it in* E, *draw* P E. *I ſay, the right Line* P E *touches the* Parabola *in the point* P *only.*

1. Let any other point as *p*, (*See Fig. p.*7.) be aſſign'd, where the Tangent P E may be ſaid to meet the Curve: 'Tis eaſie to ſhew that Suppoſition to be Impoſſible or Abſurd. For, drawing *p* F to the *Focus*, and *p a* parallel to the *Axis*, the Line *p* E is perpendicular to the Baſe of the Iſoſceles Triangle F *p* A ſince it biſſects it in E: Wherefore

fore A *p* and F *p* are equal; but *a p* = to F *p* *from the Generation of the Parabola*, therefore *a p* will be = A *p*, which is Abſurd: For the Angle *p a* A is a Right one; therefore the Line P E does notmeet the *Parabola* in the point P.

2. Again, if it be poſſible let the *Parabola* below the point P, fall without the Line E P L with reſpect to the Axis, that is, let the part P L of the Line E P L be always within the *Parabola*. From any one of the points as S *of the ſuppos'd Parabola* lying without the Line L P, having drawn S F to the *Focus*, and S V parallel to the *Axis*; by the Generation of the *Parabola*, S F = S V; but S A is greater than S V, the Angle S V A being a right One; therefore S A is greater than S F, which is Abſurd: For the Line E P L biſſects A F, and is perpendicular to it, *and likewiſe paſſes through the point S*, wherefore S A and S F will be equal. Therefore it is evident, that S is not one of the points of the *Parabola*, and conſequently that the Propoſition is true.

COR-

COROLLARY.

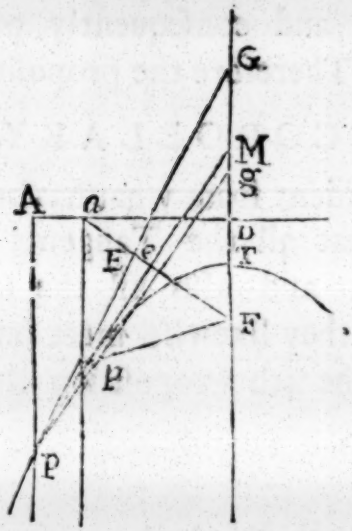
It is evident, that the Angle F G E, made by the Tangent P G and the *Axis*, is equal to the Angle F A D, which is equal to the Angle A P E or F P E; for the two Triangles F G E, F A D are Right angled at E and D, and have the Angle at F common; likewiſe the Angle at the point A of the Triangle P A E is equal to the Angle A F D, by reaſon of the parallel Lines P A, F D, and the Triangle F P E is equal and ſimilar to the Triangle A P E. It is alſo manifeſt, that the Triangle P F G is an Iſoſceles one.

PROP. V.

There can be but one Line, as P E G, *which touches the* Parabola *in the ſame point* P, *and doth not cut it.*

Suppoſe it poſſible for another Tangent as P *g* to touch it in the ſame point. From the *Focus* F having drawn F *a* perpendicular to P *g*, F *a* will meet A D in ſome point as *a*: For the Tangent P *g* always meets the *Ax-*

is



is, and can never be parallel to it, for then it would be a Diameter, and would pass within the *Parabola*: Having drawn *a p* parallel to the *Axis*, and so that it meets the *Parabola* in *p*, *p* F will be equal to *p a*, *by the Generation of the Parabola*: and F *a* being bisected in *e*, the right Line *p e* M will touch the *Parabola* in *p*, and will be parallel to P *g*, *by the precedent Proposition, being each of 'em perpendicular to* F *a*: But the Tangent P E meets the other Tangent *p e* without the *Parabola*, since both of 'em are Tangents; wherefore P *g* parallel to *p e* will pass within the *Parabola*, which is Absurd; for it was suppos'd a

Tan-

Tangent, and conſequently ought to be without: Therefore the propoſition is true.

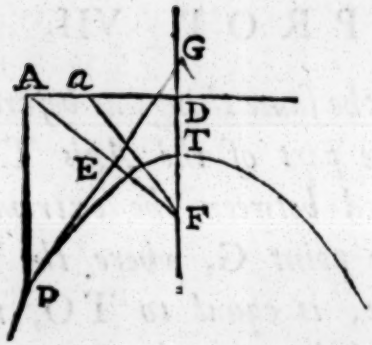
COROLLARY.

'Tis evident from what has been demonſtrated that all the Tangents meet both the *Axis*, and alſo all the Diameters, and that they likewiſe meet one another, ſince by the 4th Propoſition they are not parallel.

PROP. VI.

To find a Tangent, which on the ſame ſide with the Parabola and the Tangent, makes an Angle with the Axis leſs than any given Angle; provided the Angle given be not greater than a Right one.

Let the Line F*a* be drawn ſo, that the Angle F *a* D may be equal to the Angle given; then take the point A, on the other ſide of *a*, with regard to the *Axis*: I ſay, It is evident, that the Angle F A D will be leſs then the given Angle F *a* D; and (by *Cor. Prop.* 4.) the Line P E G drawn through the

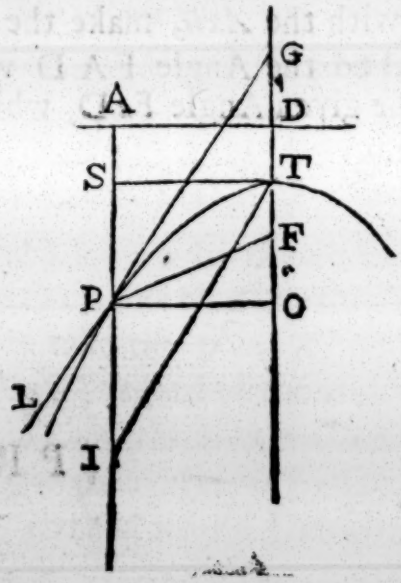


the point P, (where the Line A P parallel to the *Axis* meets the *Parabola*) and the point E (which biſſects F A) will be a Tangent to the *Parabola* in the point P; and will, with the *Axis*, make the Angle P G F equal to the Angle F A D which is leſs then the given Angle F*a* D, which was to be done.

PROP.

PROP. VII.

Supposing the same things as before; I say, that the part of the Axis TG *comprehended between the extremity* T, *and the point* G, *where the Tangent meets it, is equal to* TO, *the part contain'd between the same extremity* T, *and* O *that point where the Ordinate* PO *drawn from* P *the point of Contact of that Tangent, meets the Axis.*



By *Cor. Prop.* 4. the Triangle PGF is an Iſoſceles, therefore FG is = FP = PA (*from the Generation of the Parabola*) or DO, becauſe PO is parallel to AD; and if from the equal Lines FG, DO be taken the equal Lines FT, DT, the remaining Lines TG, TO will be equal. W. W. D.

COROLLARY.

If the Tangent TS be drawn thro' the point T, and produc'd till it meet the Diameter API; and if the Line TI be drawn parallel to the Tangent PG; it follows that PS and PI are equal, for PS is = TO, and PI = TG.

PROP.

PROP. VIII.

The ſame things remaining as before, the Angle IPL *made by the Tangent* PL, *and the Diameter* PI *drawn thro' the Point of Contact, is equal to the Angle* FPG *contain'd between the Tangent* LPG *and the Line* FP *drawn from the Focus* F *to the Point of Contact* P.

By *Cor. Prop.* 4. (*See Fig. in p.* 14.) the Angle FPG is equal to the Angle GPA which is equal to the Angle LPI; wherefore the Angle IPL is equal to the Angle FPG. W.W. D.

COROLLARY.

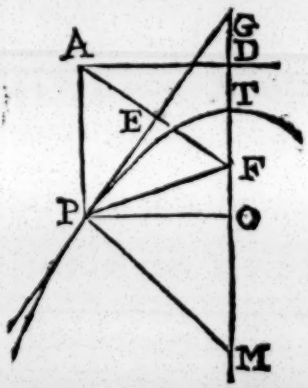
It is alſo apparent that the Angle LPF is equal to GPI, by adding the common Angle FPI to the two equal Angles IPL, and FPG.

PROP.

PROP. IX.

If thro' the Point of Contact P, P M *be drawn perpendicular to the Tangent, and continu'd till it meet the Axis in* M ; *I ſay the Line* O M *contain'd between the Foot of the Ordinate* O, *and the Point* M, *is equal to half Parameter of the Axis.*

The Line P G is perpendicular to F A, (*by Prop.* 4.) wherefore A F and P M are



parallel ; but the Triangles A D F and P O M are equal and ſimilar, by reaſon of the Parallels P A, D M, and A D, P O ; wherefore O M is equal to D F, which is

 equal

equal ½ the Parameter *by the Definition of the Parameter.* W. W. D.

PROP. X.

In the Parabola TEP, *whoſe Axis is* TO, *draw the Tangent* PH *to the Point* P, *meeting the Axis in* H, *and another Tangent, as* TB, *to the Vertex* T *meeting another Diameter as* DPB *produced to* B; *Then will the Triangle* TAH *made by the two Tangents and the Axis be equal and ſimilar to the Triangle* BAP *made by the ſame Tangents, and the Diameter* BP *produced*

Having drawn PO (*See Fig. in the twentieth Page*) at right Angles to the Axis, TO is = TH; (*by the ſeventh Prop.*) but BP is = TO, therefore BP = TH: Theſe Lines BP and TH are alſo parallel, as likewiſe are BT and PO; wherefore the Triangles TAH, and BAP are equal and ſimilar. W. W. *D.*

COROL-

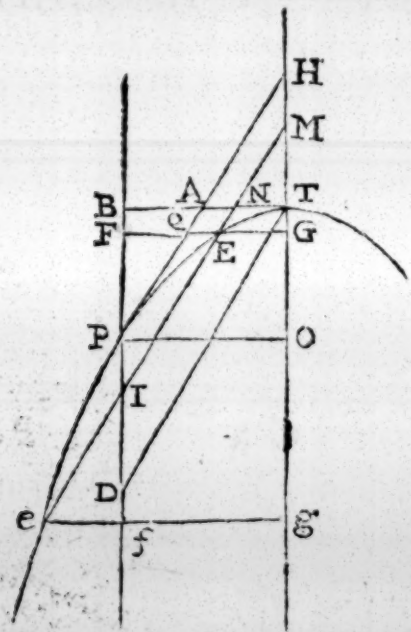
COROLLARY.

Having drawn T D parallel to P H, if each of the equal Triangles be added to the Trapezium T D P A, the Triangle T D B will be equal to the Parallelogram T D P H, which is equal to the Rectangled Parallelogram T O P B, becauſe they have equal Baſes T O, T H, and are between the ſame Parallels; but the Rectangle T O P B is yet farther equal to the Triangle P O H, by reaſon of the two equal Triangles A P B and A T H.

PROP. XI.

The same things being supposed as before; If two Lines EG *and* EM *be drawn through any point of the Parabola as* E, *parallel to the two Tangents* PAH *and* BAT; *the Triangle* EGM *is equal to the Rectangle* GTBF.

From the nature of parallel Lines, the Tri-



angle P OH. Triangle E G M :: P O. E G.

but

but (*by Prop.* 1. *Cor.* 3:) P O *q*, E G *q* :: T O. T G: And T O. T G :: Rectangle T O P B. Rectangle T G F B: therefore (by proportion of Equality) Triangle P O H. Triangle E G M :: Rectangle T O P B. Rectangle T G F B; but the Triangle P O H is equal to the Rectangle T O P B (*by Cor. Prop. Precedent*;) therefore the Triangle E G M is equal to the Rectangle T G F B. W. W. D.

After the ſame manner the Triangle *e g* M may be demonſtrated equal to the Rectangle T *g f* B.

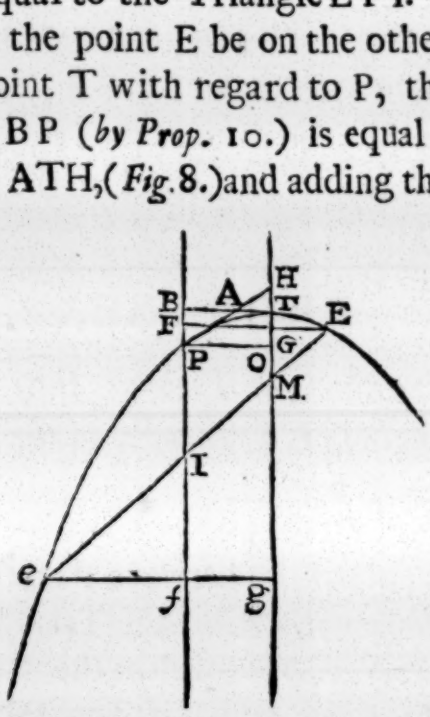
PROP. XII.

Retaining the ſame Figure (Fig. 7.) *the Triangle* E F I *is equal to the Parallelogram* P I M H.

If the point E be between P and T, (*See Fig. in the foregoing Page*) the Triangle POH (*by Cor. Prop.* 10.) is equal to the Rectangle P O T B; from which if you take equal things, *viz.* if from the Triangle P O H you take the Triangle M E G, and from the Rectangle P O T B, the Rectangle T G E B equal to the Triangle M E G: (*by*

Prop. 11.) And again, if from each you take the Quadrilateral Figure PQGO, there remains the Quadrilateral Figure E Q H M equal to the Triangle Q F P ; to each of which, if the common Quadrilateral Figure P I E Q be added, the Parallelogram PIHM will be equal to the Triangle E F I.

But if the point E be on the other hand of the point T with regard to P, the Triangle A B P (*by Prop.* 10.) is equal to the Triangle ATH, (*Fig.* 8.) and adding the com-



mon Figure PIMTA to each of them, the Parallelogram PIMH will be equal to the Trapezium I M T B, from which Trapezium if the Rectangle G T B F be taken, and to the

the remaining Figure FGMI be added to the Triangle G E M which is equal to the Rectangle G T B F; (*by Prop.* 11.) then will the Triangle E F I be equal to the Parallelogram P I M H.

Laſtly, If the Point E be on the other ſide of P as in *e*: The Triangle T A H is equal to the Triangle A B P; (*by Prop.* 10.) and if the Figure T *g f* P A be added to each, the Quadrilateral Figure H *g f* P, will be equal to the Rectangle T *g f* B, which is equal to the Triangle *g e* M (*by the precedent*); then if the common Quadrilateral Figure M *g f* I be taken from the Quadrilateral Figure H *g f* P, and the Triangle M *g e* which is equal to it; there remains the Parallelogram P I M H equal to the Triangle *e f* I. W. W. D.

PROP. XIII.

If from a Point E *in the Parabola,* (Fig. p.20.22.) *be drawn the right Line* E *e parallel to a Tangent, as* P H. *Then will the right Line* E *e meet the Parabola in another point as e; and likewiſe be biſſected in* I *by the Diameter* P I, *drawn thro' the Point of* contact

If B T be the Tangent, the *1ſt.* part of the *Prop.* is evident *from the Generation of the Parabola*: But if another Tangent be drawn as P H; A Tangent (by *Prop.* 6.) may be found, which ſhall make an Angle with the *Axis*, leſs then the Angle G H P, and on the ſame ſide of the *Axis*; wherefore this Tangent will meet the Tangent P H on one ſide of P, and the Tangent T A on the other; and the Line E *e* being Parallel to P H, will alſo meet the two Tangents either without the *Parabola* or upon it, on each ſide the Diameter P I, and conſequently the *Parabola* in two Points as E, *e*, which is the firſt part of the *Prop.*

The

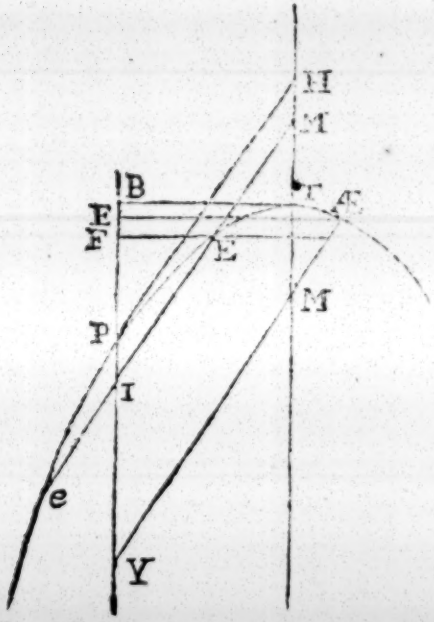
The Triangle E F I (*by prop. Preced.*) is equal to the Parallelogram P I M H, which is alſo equal to the Triangle *e f* I, wherefore the two Triangles are equal: But they are likewiſe ſimilar, by reaſon of the Parallel Lines, which form them, therefore E I is equal to *e* I, W. W. D.

DEFINITION.

Thoſe Lines are called *Ordinates* to a *Diameter* which are are drawn parallel to a Line touching the *Parabola* in the vertex of that Diameter; as thoſe Lines are Ordinates to the *Axis* which are perpendicular to it.

PROP. XIV.

The ſquares of the Ordinates as EI, ÆY *belonging to one and the ſame Diameter as* PI, *are to one another, as the Parts of this Diameter* PI, PY *compriz'd between the Vertex* P, *and the Points where it meets the Ordinates* I *and* Y.



For (the Triangles E F I, and ÆFY being ſimilar) the Triangle EFI is to the Triangle ÆFY as the ſquare of EI to the ſquare of ÆY; but (*by the precedent Prop.*) the Triangle

E F I

E F I is equal to the Parallelogram P I M H, and the Triangle Æ F Y is equal to the Parallelogram P Y M H; and theſe Parallelograms are to one another as P I to P Y; wherefore the ſquare of E I is to the ſquare of Æ Y, as P I to P Y. W. W. D.

'Tis the ſame thing if the Ordinates be taken on the other ſide the Diameter as *e* I; for they are equal to the Former. (*By Prop.* 13.)

Definition of the Parameter.

The *Parameter* belonging to any Diameter is a third Proportional to any Abſciſſa, as P I, and its reſpective Ordinate as I E.

COROLLARY.

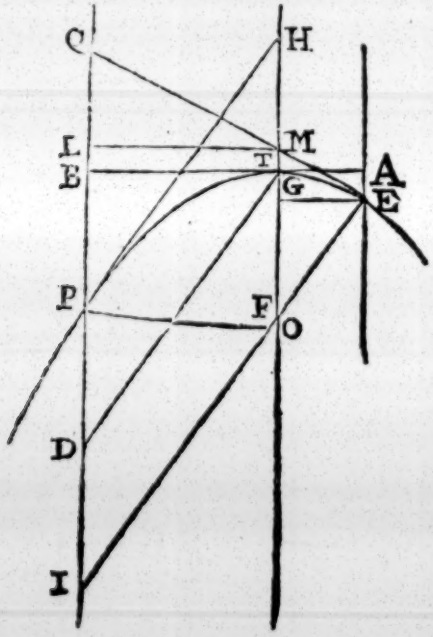
It is evident that the ſquare of an Ordinate to any Diameter (as Æ Y is an Ordinate to the Diameter P Y) is equal to a Rectangle under the Parameter, and P Y that part of the Diameter which is contained between the Vertex P, and Y the Point where the Ordinate meets the Diameter.

PROP.

PROP. XV.

If a Tangent as EQ *be drawn thro' the point* E *of a* Parabola, *meeting any Diameter as* PQ *in* Q; *and if thro' the ſame point* EFI, *be drawn an Ordinate to this Diameter, I ſay* PQ *and* PI *are equal.*

Draw the *Axis* TF, and BTA a Tangent in the Point T, and PH a Tangent in P, which PH will be parallel to EI, (by



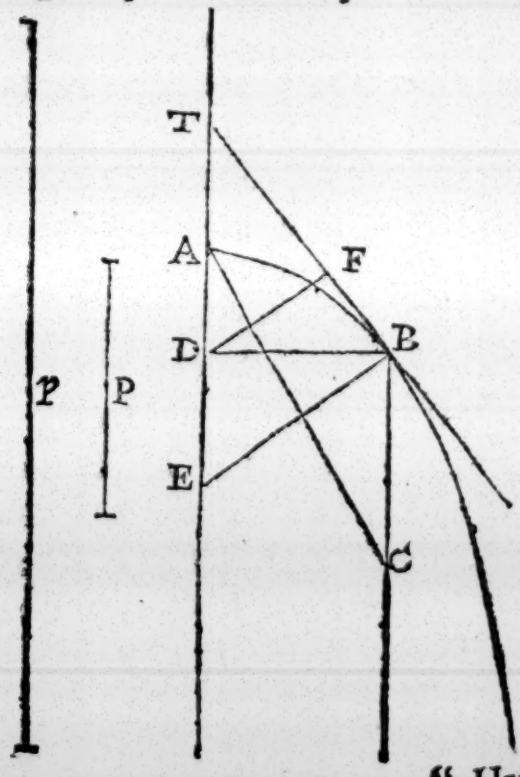
Prop.

Prop. 13.) to which E I let T D be drawn parallel, as likewiſe M L, E G, P O parallel to B A; then (by *Cor.* 3. *Prop.* 1.) the Square of P O, or B T which is equal to it, is to the Square of E G as T O to T G, or as their doubles B D to M G. And becauſe of the parallel Lines, the Square of B T is to the Square of E G, as the Square of B D to the Square of G F; therefore the Square of B D is to the Square of G F as B D is to M G, and conſequently G F is a mean Proportional between B D and M G. In like manner the Square of B T or the Square of L M is to the Square of G E as the Square of Q L to the Square of M G; wherefore the Square of Q L is to the Square of M G as B D to M G; Q L therefore is a mean proportional between the ſame B D, M G; and conſequently G F and Q L are equal; and if to theſe 2 equal Lines be added the equal Lines L B and T G, as likewiſe the equal Lines B P and T H, the 2 Summs will be equal, that is P Q will be equal to F H which is equal to P I *from the nature of Parallel Lines.* W. W. D.

PROP.

PROP. XVI.

" *In the Parabola* A F B, *whose Axis is*
" A E, *and any other Diameter* B C;
" *from the extremity* B *of the Dia-*
" *meter* B C *having drawn* B D *an Or-*
" *dinate to the Axis. I say the Pa-*
" *rameter of the Diameter* B C, *ex-*
" *ceeds the Parameter of the Axis* A E *by*
" *Quadruple the intercepted Axis* A D.



" Having

" Having drawn B T touching the *Parabola* in B, and meeting the *Axis* in T; " B E perpendicular to the Tangent in " B; D F parallel to B E; and A C parallel to B T, which A C will be an Ordinate to the Diameter B C (by *Def. Prop.* 13.); and consequently equal to T B (and B C = A T fromparallel lines) which " is equal to A D; (by *Prop.* 7.): But D E is " equal to the Semiparameter of the *Axis* " (by *Prop.* 9.)

" Let the Right Line P be equal to the " Parameter of the *Axis*, and *p* equal to " the Parameter of the Diameter B C. " Then (*Prop.* 1. and *Prop.* 14. *Cor.*) A C *q* " is to B T *q*, as the Rectangle B C × *p* " is to the Rectangle A D equal to B C × P; " but these Rectangles having equal Altitudes are to one another as *p* to P; " And, from the Similarity of Triangles, " the Square of B T is to the Square of " B D, as B T to B F; and B T is to B F " as E T to E D; But E T is equal to " D T, or D A twice taken, together with " D E, which D E is equal to $\frac{1}{2}$ P; then E T " is to E D as 2 E T to 2 E D; but 2 E T " is equal to 4 D A + P, and 2 E D = P,

" there-

" therefore E T is to E D as 4 D A + P
" is to P, and (and *ex Equo*) *p* will be to
" P as 4 D A + P is to P, consequently *p*
" is equal to 4 D A + P. W. W. D.

COROLLARY I.

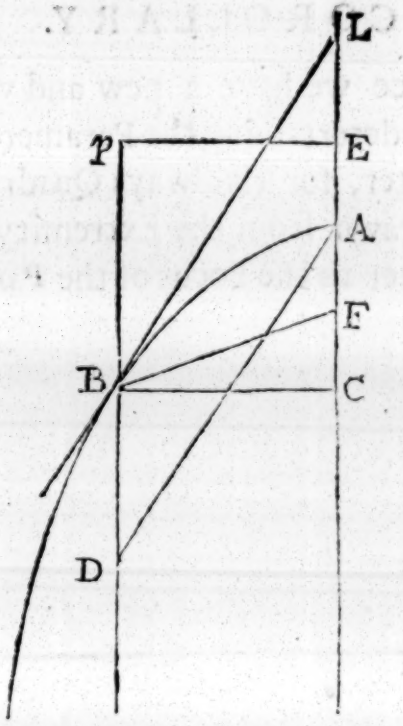
" It is evident from this Propoſition that
" the Parameters of thoſe Diameters which
" are farther diſtant from the *Axis* are
" greater then the Parameters of thoſe Di-
" ameters which are nearer to it. For the
" ſquare of B D. is to the ſquare of A C
" as the Parameter of the *Axis.* to the Pa-
" rameter of the Diameter B C.

PROP. XVII.

" *In the* Parabola B A, *whoſe* Axis *is* A
" C, *draw any Diameter as* B D, *and*
" F B *from the Focus* F *to* B, *the ex-*
" *tremity of the Diameter* B D. *And*
" *the Right Line* F B *will be equal to*
" ¼ *of the Parameter of that Diame-*
" *ter.*

" A E

" A E is equal to A F (*by the Conſtruction*
" *of the* Parabola) which is equal to $\frac{1}{4}$ of



" the Parameter of the *Axis*, and by the
" laſt *Propoſition* the Parameter of the Dia-
" meter B D exceeds the Parameter of the
" *Axis* by 4 times A C; But A E is equal
" to $\frac{1}{4}$ of the Parameter of the *Axis*;
" wherefore C E is equal to $\frac{1}{4}$ of the Dia-
" meter B D; but C E is equal to B *p*,
" which is equal to B F; therefore B F is

 equal

" equal to $\frac{1}{4}$ of the Parameter belonging
" to the Diameter B D. W. W. D.

COROLLARY.

" Hence we have a new and very easie
" way of determining the Parameter of any
" Diameter, for 'tis always Quadruple of a
" Line drawn from the Extremity of that
" Diameter to the Focus of the *Parabola*.

A TREATES OF *Conick Sections.*

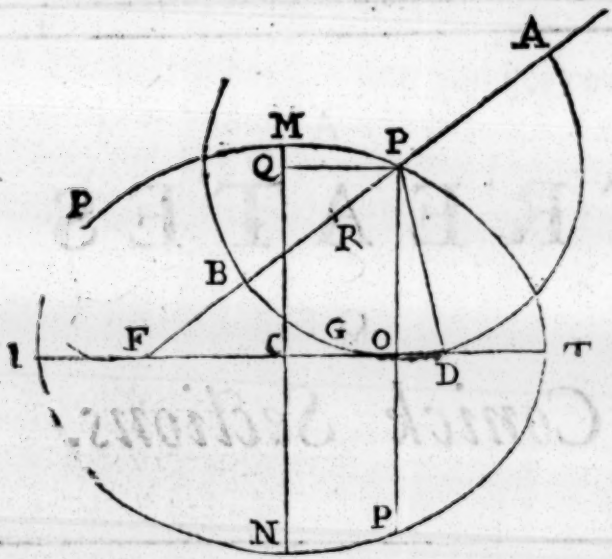
PART. II.

THE ELLIPSIS.

Of the Generation of the ELLIPSIS.

IF upon a Plain be drawn the right Line I T, and bisected in C, and the Points and D be taken at equal distances

from C. I ſay as many Points as you pleaſe of P, may be found in ſuch an order that



two Lines P F and P D, drawn from that Point P to the two Points F and D, being joyn'd together will be equal to the Line I T.

Divide I T any how into two parts, and make one of them the Radius of a Circle whoſe Centre let be F, and the other the Radius of a Circle whoſe Centre let be D, and with theſe Radij and upon theſe Centers deſcribe two Circles, which will interſect one another in 2 Points as P, P, one of which is above and the other below the Line I T, and at

equal

equal distances from it; and the Line P O P which joins the two Points P, P, will be perpendicular to I T, and after the same manner an infinite number of Points may be found as P; which was to be done.

COROL.

It is evident that A line passing through the Points P, P, will pass thro' I and T, and that the Line P T P I will include space, and likewise that all the Lines as P O P which are perpendicular to I T and terminated by the Curve Line P, P on each side of the Line I T, will be bessected in O by the same Line I T.

DEFIN.

1. The Curve Line P T P I is called an *Ellipsis*.

2. The Point C, the *Centre* of the *Ellipsis*.

3. The right Line I T, the *Transverse Axis*.

4. The right Line N C M, perpendicular to I T, passing thro' the Centre, and terminated by the *Ellipsis*, is called the *conjugate Axis*.

5. The Points F and D, are called the *Foci.*

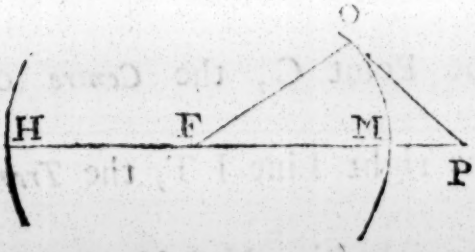
6. Right Lines drawn from the Points in the *Elipsis* perpendicular to the *Axes*, are called *Ordinates to the Axes*, as P O is an ordinate to the *Axis* I T.

7. All the right Lines which pass thro' the Centre C, and are terminated at each end by the *Elipsis*, are called *Diameters*

8. A right Line which meets the *Elipsis* only in one Point is call'd a *Tangent* to the *Elipsis* in that Point.

LEMMA.

In every Right-angled Triangle as F O P, the Rectangle made by multiplying P H, the Sum of the Hypothenuse F P, and one



side, as F O, into M P their difference, is equal to the square of the other side P O.

From

From F as a Centre and with the Radius F O, having deſcribed the Circle M O H, and drawn P F to H, the ſide P O touches the Circle in O, becauſe the Angle at O is right; wherefore the Rectangle P M multiply'd into P H, is equal to the ſquare of P O. W. W. D.

PROP. I.

The Elliptis being formed according to the precedent Method. The ſquare of any Ordinate to the Tranſverſe Axis as P O, *is to the Rectangle* I O T *under the Parts of the Tranſverſe Axis each way from the Ordinate, as the Rectangle* I F T *is to the ſquare of* I C *or* C T. *(Fig.* 1.*)*

Having prolonged F P to A, make F A equal to I T, and biſſect it in R; from the Point P as a Center, and with the Radius P D or P A equal to it, deſcribe the (*See Fig. in Pag.* 36) Circle A D B, which will meet A F in B, and I T in G, if the Points D and O are not

coincident; for ſhould they be concident, the three Points D,O,G wou'd coincide, and the Circle wou'd touch the Line I T in O. But ſuppoſing firſt, that the three Points D, O, G are not concident. From the Nature of the Circle A D B, the Rectangle F A x F B is equal to the Rectangle F D x F G, wherefore F A is to F D as F G to F B; and their halves F R or C T is to C D, as C O is to R P; *and by compoſition*, C T is to C T + C D as C O is to C O + R P, conſequently C T is to C O as C T + C D is to C O + R P; and farther *by compoſition* C T is to C T + C O (which added together are equal to I O) as C T + C D (which added together are equal to I D) is to C T + C D + C O + R P. But C T + C D + C O + R P or its equal F R + R P + F C + C O is equal to the Sum of F P and F O, wherefore C T is to I O as I D is to F P + O F.

In like manner, reaſſuming that Proportion above of C T, to C D as C O to R P, and by *Diviſion*, C T is to C T — C D, as C O to C O — R P; and conſequently, C T is to C O as C T — C D is to C O — R P and farther by Diviſion, C T is to C T — C O (which is equal to O F) as C T — C D

CD (which is equal to DT) is to CT — CD — CO + RP. But CT — CD — CO + RP, or its equal FR + RP — FC — CO, is equal to the Difference of FP and FO; wherefore CT is to OT as DT to the Difference of FP and FO.

Now if the Proportion found above at the end of the laſt Section ſave one, and that which we found laſt be multiply'd one into another Antecedent into Antecedent and conſequent into conſequent, we ſhall have the Square of CT to the Rectangle IOT as the Rectangle IDT to the Rectangle FP + FO × FP — FO equal (*by the preceding* Lemma) to the Square of PO. W. W. D.

If the Points D, G, O coincide, the Demonſtration will ſtill be the ſame, for FG, FO, and FD will be equal. Which cannot make any Alteration.

PROP.

PROP. II.

Retaining the ſame Figure; the Rectangle I D T *is equal to the Square of* C M, *which is* ½ *of the Conjugate Axis.*

Suppoſe M C be an Ordinate, then (*by the foregoing Prop.*) the Square of C T is to the Rectangle I C T *i. e.* the Square of C T, as the Rectangle I D T, is to the Square of M C. Therefore the Square of M C is equal to the Rectangle I D T or I F T which is equal to it. W. W. D.

PROP. III.

Still retaining the ſame Figure. The Square of the Conjugate Axis N M, *is to the Square of the Tranſverſe Axis* I T, *as the Square of* P O *an Ordinate to the Tranſverſe, to the Rectangle* I O T.

The Square of C T (*Prop.* 1.) is to the Rectangle I D T which is equal (*by the preceding Prop.*) to the Square of C M, as the Rectangle

Rectangle I O T to the Square of P O; but the Square of I T is Quadruple the Square of C T, and the Square of N M, is alſo Quadruple the Square of C M; therefore the Square of I T is to the Square of N M as the Rectangle I O T to the Square of P O. W. W. D.

PROP. IV.

Still retaining the ſame Figure as above, the Square of P Q *which is an Ordinate to the Conjugate Axis, is to the Rectangle* N Q M, *as the Square of the Tranſverſe Axis* I T, *is to the Square of the Conjugate Axis* N M.

By the 1 *and* 2 *Propoſition*, the Square of C T is to the Square of C M as the Rectangle I O T (which is equal to the Square of C T leſs the Square of C O) is to the Square of P O; and by *Diviſion* the Square of C T, is to the Square of C T leſs the Square of C T more the Square of C O; as the Square of C M is to the Square of C M — the Square of P O, and by the Preceeding Propoſition

on, The Square of I T is to the Square of N M, as the Square of C O or P Q is to the Rectangle N Q M, (which is equal to the Square of C M leſs the Square of P O or C Q. W. W. D.

DEFIN.

A third proportional to the two *Axes* is call'd the *Parameter* of that *Axis* which ſtands firſt in the Proportion, thus if the Tranſverſe *Axis* I T, be to the Conjugate N M, as the Conjugate N M is to a right Line Y T, this Y T is called the Parameter of the Tranſverſe; but if N M (*See Fig. in Pag.* 45.) be to I T as I T to another Line, that Line will be the Parameter of the Conjugate.

2. The *Figure* of an Axis, is a Rectangle under that Axis and its Parameter, as the Figure of the Tranſverſe I T is the Rectangle I T Y.

COROLLARY.

It is evident, that the Square of one of the Axes, is equal to the Figure of the other Axis.

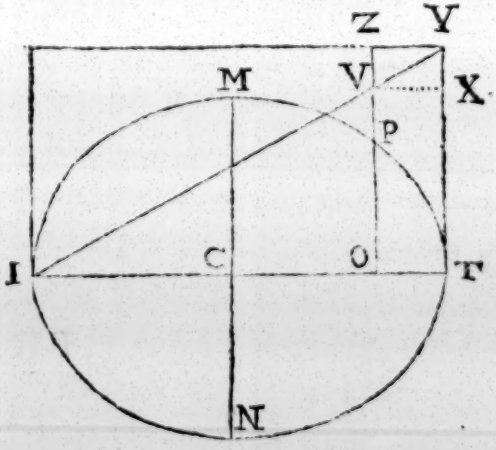
PROP.

Edward Simpson

PROP. V.

Supposing the Ellipsis *constructed as above, the Square of* PO, *an Ordinate to the Axis* IT, *is equal to the Rectangle* OTXV *under* TY *the Parameter belonging to this Axis, and* OT *the part comprised between the Extremity* T *and* O, *the Point where the Ordinate meets the Axis, less the Rectangle* VXYZ *which is similar, and similary posited to the Rectangle* IY *(the Figure of* IT.*)*

The Square of NM *(by Prop.* 3 and 4.*)*



is to the Square of IT, as PO *q* is to the Rectangle

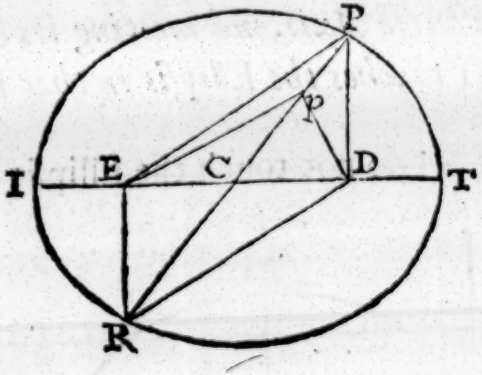
Rectangle I O T; but by *the Property of the Parameter*, the Square of N M is to the Square of I T, as Y T is to I T; and as Y T is to I T ſo alſo is V O to I O; or rather the Rectangle V O T to the Rectangle I O T, having T O for their common Altitude, and (*by proportion of Equality*) the Square of P O is to the Rectangle I O T as the Rectangle V O T X is to the Rectangle I O T: Wherefore the Square of P O is equal to the Rectangle O T X V, which is leſs than the Rectangle O T Y Z by the Rectangle V Y, which V Y is ſimilar and ſimilarly poſited to the Figure I Y W. W. D.

PROP.

PROP. VI.

All the Diameters as P R *are biſſected in the Center* C.

If it be poſſible for C P to be greater



than C R, make C *p* equal to C R, and having drawn to the Foci the lines E *p*, D *p*, E R, D R, and E P, D R; the two Triangles C D *p* and C E R will be equal and ſimilar; as likewiſe will the Triangles C D R and C E *p* for C D, C E, and C R, C *p* are equal, wherefore E R is equal to D *p*, and D R to E *p*; the Summ therefore of E *p*, D *p* will be equal to I T, for the ſumm of E R, D R is equal to it (*from the Generation of the Ellipſis*); but the ſumm

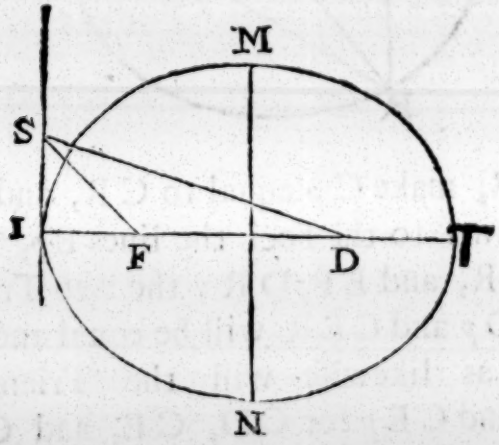
of

of EP, DP is also equal to I T, therefore the summ E*p*, D*p* is equal to the Summ of EP, DP, which is absurd. Wherefore the Proposition is true.

PROP. VII.

A right line as SI *being perpendicular to the Transverse Axis, and meeting its extremity* I *touches the Ellipsis in that Point.*

For if S I do not touch the Ellipsis in the



Point I only, but meet it in some other Point as S; having drawn the lines FS, DS, to the Foci; *from the Generation*, the Lines F S and D S added together will be equal

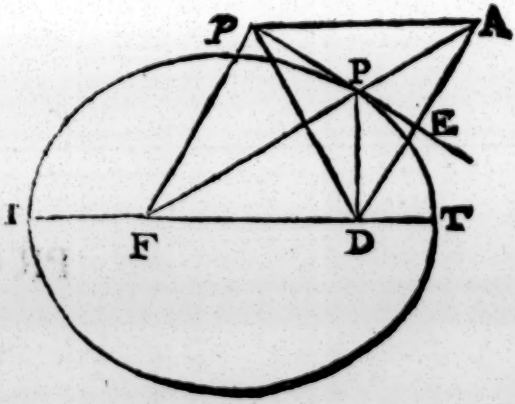
to

to I T; and becauſe the Triangle F I S is right Angled at I, F S is greater than F I, wherefore D S muſt be leſs than T F or D I which is abſurd, for D S ſubtends the right Angle I in the Triangle D I S. Therefore the Prop. is true.

PROP. VIII.

The ſame things being ſuppoſed as in the firſt Prop. draw the Line D A *and biſſect it in* E, *and then the Line* P E *will touch the Ellipſis in the Point* P.

If P E do not touch the Ellipſis in the



Point P, it will meet it in ſome other point

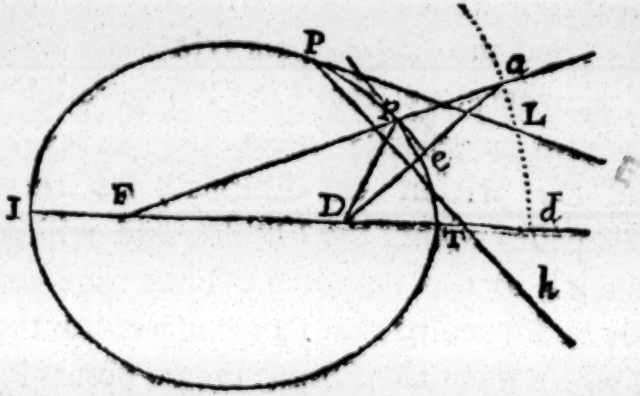
as p; by the Generation Fp, pD added together will be equal to FP and PD added together, which is equal to FA; but pD is equal to pA, for pE is perpendicular to AD, and biſſects it; wherefore Fp and PA together will be equal to FA which is abſurd; for the two ſides of the Triangle FpA can never be equal to the other ſide FA; therefore the line PE touches the Ellipſis in P. W. W. D.

PROP.

PROP. IX.

In an Ellipſis, as IPT, *there cannot be drawn above one Line, as* PE; *which will touch the Ellipſis in the ſame Point* P.

Suppoſe it be otherwiſe, and that the



Line P *b* touch the Ellipſis at the Point P. From the Focus D, having drawn D *a* perpendicular to P *b*, and from the Center F and with the Radius F *d* equal to IT, having deſcrib'd the Circle *d a* meeting D *a* in *a*, and having biſſected D *a* in *e*, draw F *a* meeting the Ellipſis in any point as *p*,

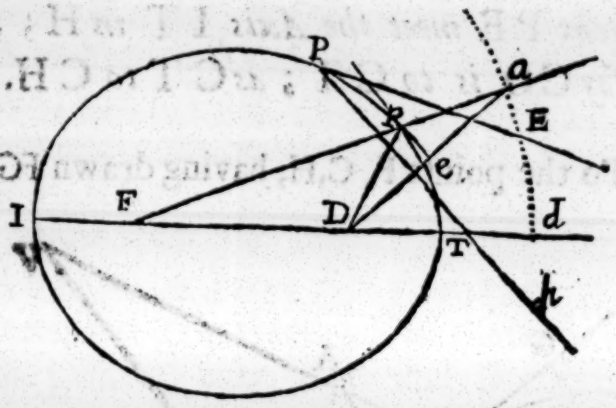
then draw D *p* and *p e*, which, passing thro' *e* the point of bisſection of the Line D *a*, will be perpendicular to D *a*, for D *p* and *p a* are equal by the *Generation of the Ellipſis*, wherefore the line *p e* will touch the Ellipſis (*by the precedent Propoſition*) at the point *p*; but P *h* is alſo perpendicular to D *a*, wherefore P *h* and *p e* will be parallel; and the Tangents P *h* and *p e* can never meet one another but without the Ellipſis; and the *Ellipſis*, paſſes thro' the Points P, *p*, wherefore the Line P *h* parallel to *p e* will neceſſarily meet the Lines F *p* and D *p* within the Ellipſis; P *h* therefore paſſes within the Ellipſis, and will not touch it as was ſuppoſed: That Line cannot be joyned to the Tangent *p e*; for then it wou'd meet the Ellipſis in two points P, *p*, therefore there can be but one Line, which ſhall touch an Ellipſis, in the ſame Point.

PROP.

PROP. X.

The same things being supposed: The Angles F *p* P, *and* D *p e, made by the Tangent p e, and the Lines p* F *and p* D *drawn from p, the point of Contact to the two Foci, are equal.*

The Triangle D *p a* is Isoscelar, and *p e*



bisects the Base, wherefore the Angle D *p e* is equal to the Angle *a p e*; but the Angle F *p* P, is equal to the Angle *a p e*; therefore F *p* P, is equal to D *p e*. W. W. D.

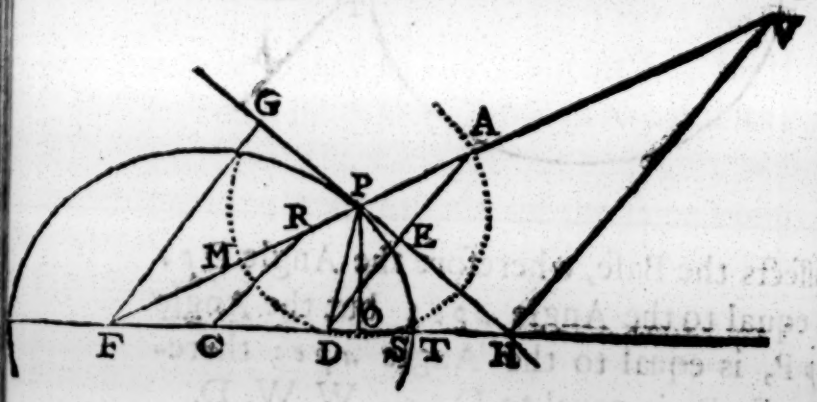
COROLLARY.

If the common Angle F *p* D be added to the two equal Angles, it is evident, that the Angle *e p* F will be equal to the Angle D *p* P.

PROP. XI.

Supposing still the same things, if the Tangent P E *meet the Axis* I T *in* H; *I say* C O *is to* C T; *as* C T *to* C H.

To the points F, C, H, having drawn FG,



CR, and H V parallel to D A; F A will be bisected in R, seeing F D is bisected in C

C: From similar Triangles F P, is to A P; as F G is to A E or its equal D E, and F G is to D E, as F H to D H; and F H is to D H, as F V to V A; wherefore *ex æquo* F P is to P A; as F V to V A; and by *Diviſion* F P — P A, (which is equal to twice R P) is to P A :: as F V — V A (which is equal to F A) is to V A; but 2 R P is to F A; as their halves *viz.* R P is to R A; wherefore *ex æquo* R P. P A :: R A. V A. By compoſition, R P. R P + P A (= R A) :: R A. R A + V A = R V. Therefore the Lines R P, R A, and R V are continual Proportionals, wherefore the Square of R A is equal to R P × R V.

Now from the Nature of the Circle, A S D M, *as in the firſt Propoſition*, which meets the Tranſverſe I T in D and S, or in the point O only, if O and D coincide; then F D will be to F A as F M to F S; and their halves C D. R A :: R P. C O. But C D. R A :: C H. R V, wherefore *ex æquo* C H. R V :: R P. C O, and the Rectangle of the Extreams and means being equal, C H × C O = R P × R V, but in the preceding Article R P × R V was demoſtrated equal to the Square of R A, which R A is

equal to C T; for *from the Generation of the Ellipsis*, F A = I T; wherefore C O. C T :: C T. C H. W. W. D.

COROLLARY.

Seeing the Lines C O, C T, C H are in continual Proportion, by inverting the Method of the Demonstration of the first Article of this Proposition, 'twill be easy to demonstrate, that I O is to O T as I H to H T; for having found that F V. V A :: F P. P A, 'tis demonstrable that R P, R A, R V will be in continual Proportion.

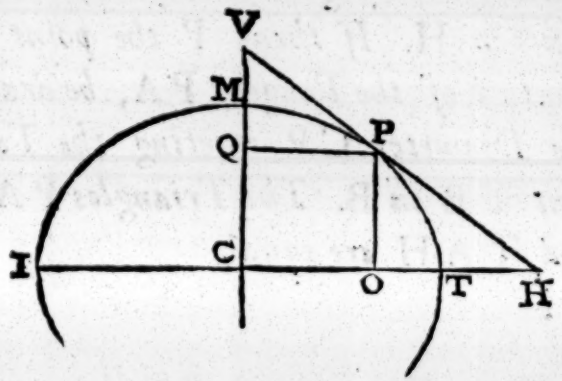
The Line I H is said to be Harmonically divided in the Points O and T.

PROP.

PROP. XII.

The ſame things being always ſuppoſed. If the Tangent P E *meet the conjugate Axis in* V, *having drawn* P Q *an Ordinate to this Axis: I ſay* C Q. *is to* C M :: *as* C M *is to* C V.

It is evident by the foregoing Prop. that



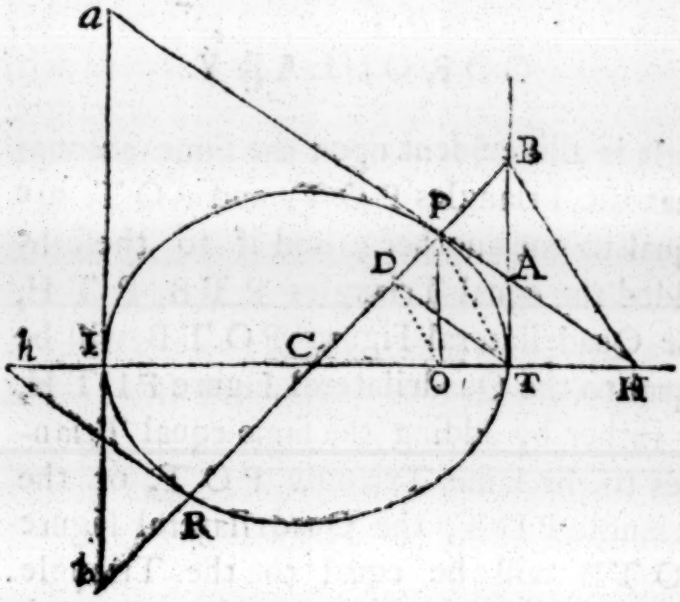
the Square of C O, is to the Square of C T :: as C O, to C H; but (by Prop 4.) C O q or Q P q. C T q :: C M q — C Q q = Rectangle N Q M. C M q; and from ſimilar Triangles C O. C H :: V Q. C V, wherefore *ex æquo* V Q. C V :: C M Q — C Q q. M C q,

CMq. By diviſion CV. CV — VQ = CQ :: CMq. CMq — CMq + CQq, *i. e.* CV. CQ :: CMq : CQq, wherefore CQ. CM :: CM, CV. W. W. D.

PROP. XIII.

In the Ellipſis IPT, *the Axis whereof is* IT, *and* BT *a Tangent to it at the extremity* T, *and* PA *another Tangent meeting the Tangent* BT *in* A, *and the Axis in* H. *If thro'* P *the point of Contact of the Tangent* PA, *be drawn the Diameter* CP *meeting the Tangent* TB *in* B. *The Triangles* PAB *and* TAH *are equal.*

From the point T having drawn T D parallel to the Tangent P H, and P O an Ordinate to the Axis; *from the Nature of Parallels*, C D. C P :: C T. C H. But (*by*



Prop. 11.) C T. C H :: C O. C T. And C O. C T :: C P. C B, wherefore (*ex æquo*) C D. C P :: C P. C B, and for the ſame reaſon, C D. C P :: C O. C T, in like manner C P. C B :: C T. C H. Therefore the Lines D O, P T, B H will be parallel to one another, and the Triangles P T B and P T H, which have a common Baſe upon one

one of the Parallels, and their Altitudes terminated by another will be equal to one another; from which ſubduct the common Triangle P A T, and the Remainders P A B and T A H will be equal. W. W. D.

COROLLARY.

It is alſo evident upon the ſame account that the Triangles P D T, and P O T are equal to one another; and if to theſe be added the equal Triangles P T B, P T H, the Quadrilateral Figure P O T B will be equal to the Quadrilateral Figure P D T H, or rather by adding the ſame equal Triangles to the ſame Triangle P O T, or the Triangle P D T, the Quadrilateral Figure P O T B will be equal to the Triangle O P H, or the Quadrilateral Figure P D T H equal to the Triangle D T B. It is likewiſe apparent that the Triangle C T B is equal to the Triangle C P H.

COR-

COROLLARY II.

Having drawn the Tangent I *b*; it is evident that the Triangles P *a b*, I *a* H are equal; for I *b* is parallel to BT, wherefore the Triangles C I *b*, C T B are ſimilar, and (the Side C I in one being equal to the Homologous Side C T in the other) are equal too: therefore the Triangle C I *b* will be equal to the Triangle C P H, which is equal to the Triangle CT B, and if to theſe equal Triangles be added the common Quadrilateral Figure I C P *a*, the Triangles P *a b*, I *a* H will be equal. But it is further evident, that what has been demonſtrated in the Triangles comprehended between the Lines C T and C P, may after the ſame manner be demonſtrated of thoſe comprehended between C I and C R, for the Tangent in the point R muſt be parallel to P H touching the Ellipſis in P; for the Triangle C P H is equal to the Triangle C T B which is equal to the Triangle C I b. And ſuppoſing that the Tangent in R meet the Axis in *h* (which point is without the Figure) 'twill be eaſy to demonſtrate after the ſame manner as above that

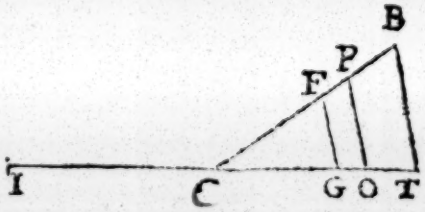
that the Triangle C I *b* is equal to the Triangle C R *b*, wherefore *ex æquo* the Triangle C R *b* is equal to the Triangle. P C H; but in these equal Triangles the Angle R C I is equal to the Angle P C H, and C R one of the sides which comprehends the equal Angle is equal to C P of the other (by *Prop* 6.) wherefore the Triangles are similar, but subcontraryly posited, R *b* therefore is parallel to P H its Homologous Side.

PROP.

LEMMA.

Let there be a Triangle, as CTB, *whose ſide* CT *being prolonged to* I, *let* CI *be equal to* CT; *from the Points* O, G, *taken at pleaſure in the ſide* CT, *drawing* OP, GF *parallel to* TB; *I ſay the Rectangle* IOT, *is to the Rectangle* IGT :: *as the Quadrilateral Figure* OTBP, *to the Quadrilateral Figure* GTBF.

CT*q*. CO*q* :: Triangle CTB. Tri-



angle COP, and by diviſion CT*q* — CO*q*. CT*q* :: Triangle CTB — Triangle COP. Triangle CTB, after the ſame manner CT*q*. CG*q* :: Triangle CTB. Triangle CGF, and by Diviſion CT*q*. CT*q* — CG*q* ::

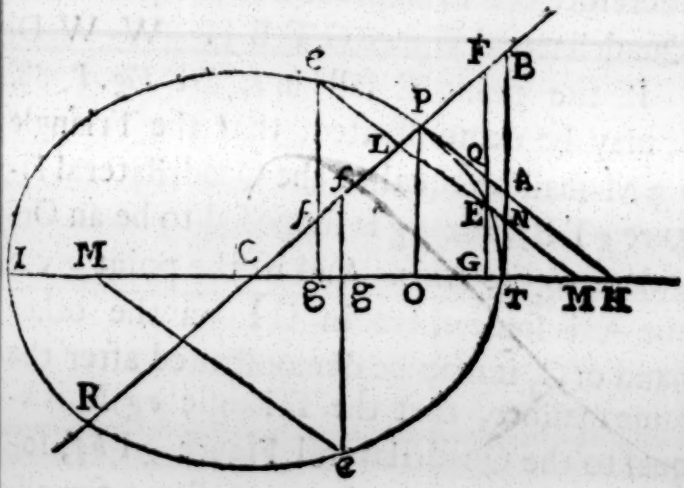
CGq :: Triangle CTB . Triangle CTB — △ CGF; wherefore *ex æquo* CTq — COq. CTq — CGq :: Triangle CTB — Triangle COP (which is equal to the Quadrilateral Figure OTBP) Triangle CTB — Triangle CGF (which is the Quadrilateral Figure GTBF) But CTq — COq is equal to the Rectangle IOT, and likewiſe CTq — CGq is equal to the Rectangle IGT ſeeing IT is divided into equal parts at the point C; wherefore the Rectangle IO·T Rectangle IGT :: Quadrilateral Figure OTBP. Quadrilateral Figure GTBF. Which was to be proved.

PROP

PROP. XIV.

If from any point E *in the Ellipsis, be drawn* LEM *parallel to the Tangent* PH, *and* FEG *parallel to the Tangent* BT, *I say the Triangle* EGM *is equal to the Quadrilateral Figure* GTBF.

By reason of the parallel Lines, the Tri-



angle POH. Triangle EGM :: PO q.
F EG q;

E G *q*; But it is evident by the 3d. Prop. that P O *q* - E G *q* :: Rectangle I O T. Rectangle I G T; and becauſe I T is divided into two equal parts in the Point C, (*by the Lemma*) the Rectangle I O T. Rectangle I G T :: Quadrilateral Figure O T B P. Quadrilateral Figure G T B F, and *ex æquo* the Triangle P O H. Triangle E G M:: Quadrilateral Figure O T B P : Quadrilateral Figure G T B F: but (by Cor. 1. Prop. preced.) the Triangle P O H is equal to the Quadrilateral Figure O T B P: therefore the Triangle E G M is equal to the Quadrilateral Figure G T B F. W. W. D.

If the point E fall in *e*, *See Fig. P. 65.* it may be demonſtrated, that the Triangle *e g* M ſhall be equal to the Quadrilateral Figure *g* T B *f*, for *e g* is ſuppoſed to be an Ordinate to the Axis. And if the point *g* cut the Axis ſomewhere in C I on the other hand of C, it may be demonſtrated after the ſame manner, that the Triangle *e g* M is equal to the Quadrilateral Figure *g* I *b f*, for the ſame demonſtration will ſerve for all, onely by conſidering what was ſaid in the 2 Corol. of the preceding Prop.

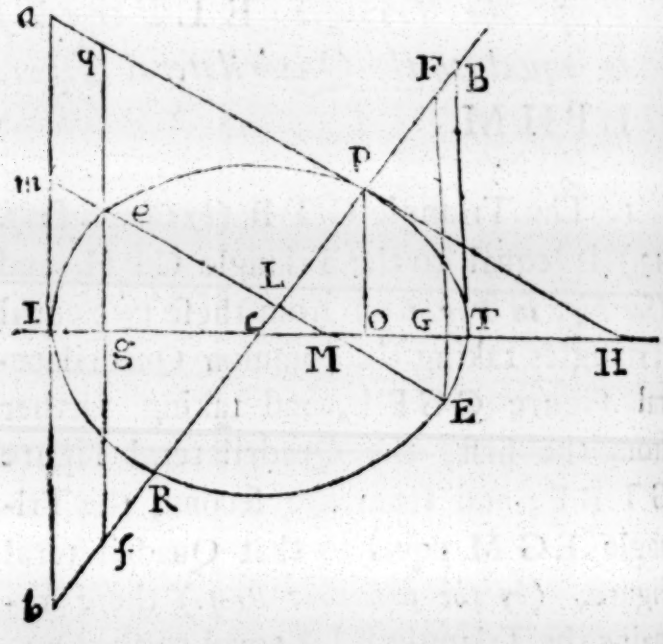
PROP.

PROP. XV.

Retaining the firſt Figure in the laſt Prop. the Triangle E L F *or* e L *f is equal to the Quadrilateral Figure* L P H M.

1. The Triangle C T B (*by Cor.* 1. *Prop.* 13.) is equal to the Triangle C P H, and (*See Fig. in Pag.* 65.) from theſe two equal Triangles taking the common Quadrilateral Figure C G E L, and taking farther from the firſt, the Quadrilateral Figure G T B F; and from the ſecond, the Triangle E G M equal to that Quadrilateral Figure, (*by the precedent Prop.*) there remains the Triangle E L F equal to the Quadrilateral Figure L P H M. W. W. D.

2. If the point E be on the other ſide of T with regard to P, and the point L fall al-



ways upon C P; from the equal Triangles C T B, C P H ſubducting the common Triangle C M L, and from the firſt remainder, taking the Quadrilateral Figure G T B F, and adding the Triangle E G M which is equal to it, we ſhall have the Triangle E L F equal to the Quadrilateral Figure L P H M.

3. But

3. But if the point E be in *e* on the other ſide of P with regard to T, (*Fig. Prop.* 14.) and the Ordinate *eg* fall upon C T; the Triangle *e g* M (by Prop. 14.) is equal to the Quadrilateral Figure *g* T B *f*, and if from theſe equals, the common Figure *f g* G E L be taken, and farther the Triangle E G M be taken from the firſt, and the Quadrilateral Figure G T B F equal to the Triangle E G M be taken from the ſecond, there remains the Triangle *e f* L = E F L; but the Triangle E F L from what was demonſtrated above is equal to the Quadrilateral Figure L P H M, wherefore the Triangle *e f* L is equal to L P H M.

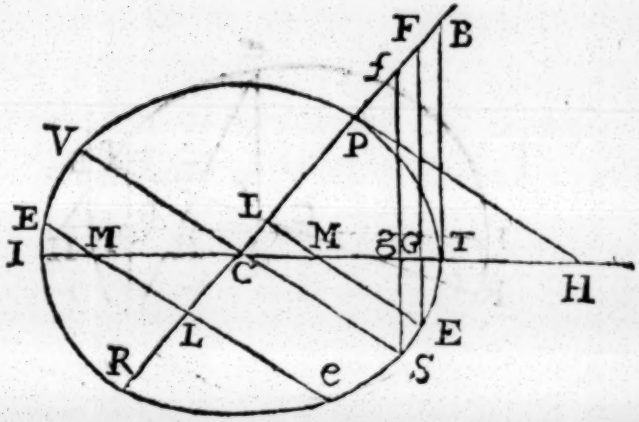
But if in the precedent caſe the point M fall upon the Axis between the Center and the Vertex T, the Triangle (*See Fig. P.* 72.) *eg*M being equal to the Quadrilateral Figure *g*TB *f*; if the common Quadrilateral Figure *fg*ML be taken from theſe equals, there will remain the Triangle *e* L *f* equal to the Quadrilateral Figure L M T B, from which if

G T B F be taken, and the Triangle E G M which is equal to it be added, the Triangle E L F will be equal to the Quadrilateral Figure L M T B; and conſequently the Triangle *e* L *f* which is equal to the Triangle E L F is equal to the Quadrilateral Figure L P H M.

In the laſt place, if the point *g* fall upon C I; (ſee *Fig. Pag.* 68.) the Triangle *a* P *b* is equal to the Triangle *a* I H; (*By Cor.* 2. *Prop.* 12.). and if from theſe 2 equal Triangles be taken the common Figure *a* P L *e g* I *a*, and farther from the firſt the Quadrilateral Figure *g* I *b f*, and from the ſecond the equal Triangle *e g* M (by *Prop.* 14.) there remains the Triangle *e* L *f* equal to the Quadrilateral Figure L P H M. W. W. D.

For all the other caſes, where the point L falls upon C R, the demonſtration will be the ſame, as any one will eaſily perceive, that conſiders what we have ſaid in *Prop.* 12. *Cor.* 2. and if the point L fall in C, the point E will be in S or in V, (the line V C S being parallel to P H) and then the Triangle C *f* S will be equal to the Triangle

C P H

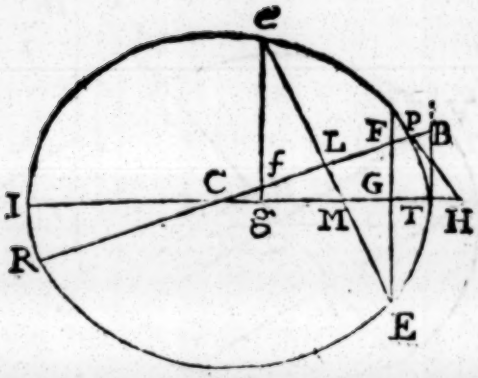


C PH; which I think is more then ſufficient for the demonſtration of this Propoſition.

PROP. XVI.

In an Ellipſis whoſe Axis is I T, *all the Right lines as* E e *parallel to a Tangent as* P H, *which meets the Axis in* H, *are cut into two equal parts in* L, *by the Diameter* R C L *drawn thro'* P *the point of Contact.*

Having placed all things as in the for



mer Propoſition, it is evident by the ſame Propoſition that the Triangles E L F, e L f are equal to one another, for each of them is equal to the Quadrilateral Figure L P H M, and they are alſo ſimilar by reaſon of the Parallels that compoſe 'em, wherefore E L = e L. W. W. D.

PROP.

PROP. XVII.

The ſame things being ſtill ſuppoſed; If the Diameter V C S *be drawn parallel to the Tangent* P H; *I ſay as* V S *Square. is to* R P *Square* :: *ſo is* E L *Square. to the Rectangle* R L P.

The Triangle C S*f* is equal (by Prop. 14.) (*See Fig. in Pag.* 71.) to the Triangle C P H; and the Triangle L E F is equal to the Quadrilateral Figure L P H M; and the Triangles C S*f*, L E F are ſimilar; wherefore they are to one another C S*q*. L E*q*; therefore C S*q*. L E*q* :: Triangle C P H. Quadrilateral Figure L P H M; but becauſe R P is biſſected in C, the Triangle C P H. Quadrilateral Figure L P H M :: C P*q*. Rectangle R L P (by Lemma Prop. 14.) wherefore C S*q*. L E*q* :: C P*q*. Rectangle R L P; or rather C S*q*. C P*q*, or their Quadruples V S*q*. R P*q* :: L E*q*. Rectangle R L P.

If the point E be in *e*, and L fall upon C R, prolong *e* L to E; it may be demonſtrated

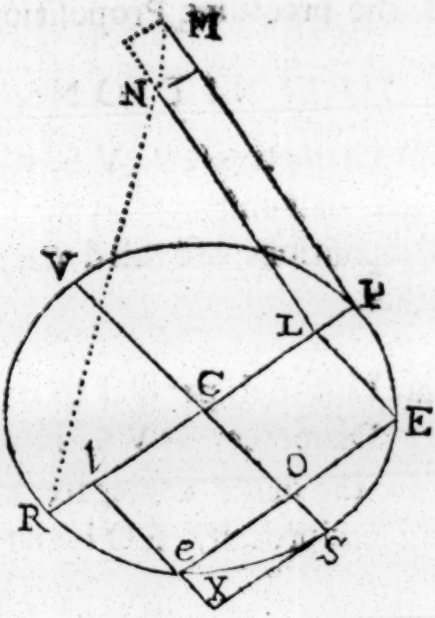
ſtrated after the ſame manner, that L E q : Rectangle R L P :: as V S q : R P q ; but, (*by the precedent Prop.*) e L = E L ; therefore the Propoſition is true.

PROP. XVIII.

Still ſuppoſing the ſame things; If the right line E e *be drawn parallel to the Diameter* R P, *I ſay that* E e *is biſſected in* O *by the Diameter* V S, *and that the Square of* R P *is to the Square of* V S, *as the Square of* E O *is to the Rectangle* V O S.

Having drawn E L and el parallel to V S (by the foregoing Propoſition) E L q : Rectangle R L P :: el q : Rectangle R l P ; but E L = el, wherefore the Rectangle R L P equal Rectangle R l P, and conſequently the lines C L, C l are equal, as alſo E O, e O which are equal to 'em ; which was the firſt thing propoſed.

Secondly, By the preceding Prop. C P q : C S q :: C P q — C L q : = Rectangle R L P



R L P : L E q ; by diviſion C P q : C P q — Rectangle R L P = C L q or E O q : : C S q : C S q — L E q or C O q : (which is equal to the Rectangle V O S) therefore C P : C S q : : or their Quadruples R P q : V S q : : O E q : Rectangle V O S. W. W. D.

C O R O L.

It is evident that the line S X parallel to O E, and drawn thro' S the Extremity of Vertex of the Diameter v S touches the Elli-

Ellipſis in the point S, which is the Converſe of the preceding Propoſitions.

DEFINITIONS.

1. The Diameters R P, V S, whoſe Properties are demonſtrated in the two foregoing Propoſitions are call'd *Conjugates* to one another.

2. The Line E L parallel to one of the conjugate Diameters as V S, is called an *Ordinate* to the other conjugate Diameter R P; and Reciprocally E O is an Ordinate to V S.

3. If you make as R P : V S :: V S : P M; this third proportional P M is called the *Parameter* of the Diameter R P; and after the ſame manner for the Diameter V S; from whence it is apparent, that the two conjugate Diameters are mean Proportionals between their Parameters.

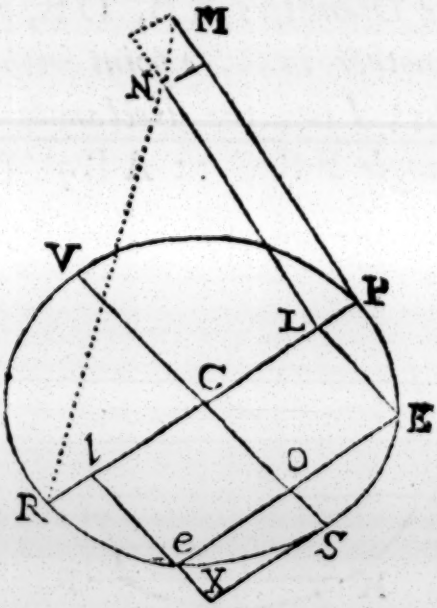
4. The Rectangle contain'd under a Diameter R P and its Parameter P M, as R M, is called the *Figure* of the Diameter R P.

PROP

PROP. XIX.

The Square of E L *an Ordinate to the Diameter* RP *is equal to the Rectangle* PN, *apply'd to the Parameter of this Diameter, which hath* P L *for its Altitude, and is less then the Rectangle* L M, *by the Rectangle* N M *which is similar to, and posited similar to the Figure* R M.

By either of the two foregoing Proposi-

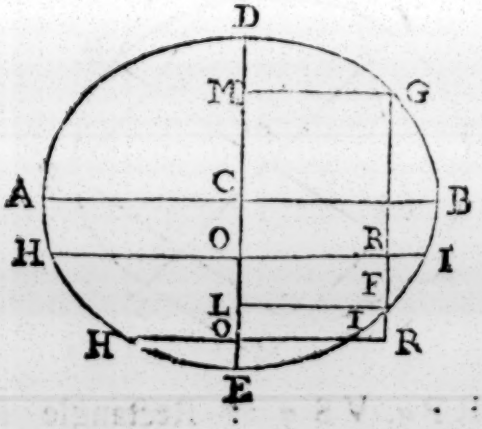


tions, RP*q*. VS*q* :: Rectangle R L P L E*q*: But RP*q*. VS*q* :: RP. PM, and RP.

RP. PM :: RL. LN; but Rectangle RLP. Rectangle NLP :: RL. NL; wherefore Rectangle RLP. LEq :: Rectangle RLP. Rectangle NLP; the Square of the Ordinate therefore is equal to the Rectangle NLP. W.W.D.

PROP. XX.

In the Ellipsis ADBE, *if two right Lines* HI, FG *be drawn parallel to two Conjugate Diameters* AB, DE, *meeting one another in* R, *a point without the Ellipsis; I say, the Rectangle* HRI. *Rectangle* FRG :: ABq. DEq.



In

Having drawn F L, G M Ordinates to the Diameter E D, they will be parallel to A B; (by *Prop.* 16 *or* 17.) and L F q : O I q :: Rectangle E L D : Rectangle E O D, (by the ſame *Prop.*) and if the point R be without the Ellipſis; by Diviſion L F q — O I q = Rectangle H R I : O I q :: Rectangle E L D — Rectangle E O D = Rectangle L O M or F R G : E O D. But if the point R be in the Ellipſis; by Diviſion O I q — L F q or O R q = Rectangle H R I : O I q :: Rectangle E O D — Rectangle E L D = Rectangle L O M or F R G : Rectangle E O D; But O I q : (by *Prop.* 16, 17.) Rectangle E O D :: A B q : E D q, wherefore the Rectangle H R I : Rectangle F R G :: A B q : E D q. W. W. D.

A TREATISE OF *Conick Sections.*

PART. III.

THE HYPERBOLA.

The *Generation of the HYPERBOLA.*

If upon a Plain the Right Line IT *be drawn, and biſſected in* C, *and the Points* F *and* D *be taken at equal Diſtances from* C *in the ſame Line produc'd both ways; you*

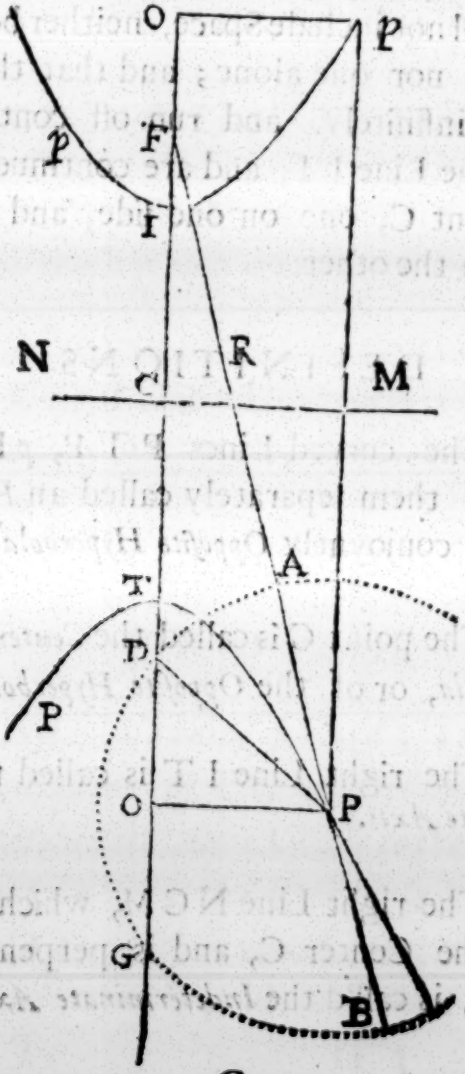
may find as many points as you pleaſe, as P *p ſo diſpoſed, that the right Line* P F *ſhall exceed* P D *by the line* I T, *or p* D *ſhall exceed p* F *by the ſame line* I T.

TAKE F P as great as you pleaſe, provided it be not leſs then F T, and then the difference betwixt F P and I T will be always greater or at leaſt equal to D T; wherefore if the point F be made the Cen ter of a Circle, whoſe Radius let be F P, and D the Center of another Circle, whoſe Radius let be D P, theſe two Circles will neceſſarily interſect one another in two Points, as P, on each ſide the line I T; or at leaſt will touch one another in the point T, in caſe P and T coincide, for then F T and D T will be the two Radij; and thus you will find different Points as P, if you take the lines F P different from one another: After the ſame manner may be found as many Points *p* as ſhall be deſired.

COROL.

It is evident by *this Generation*, that the right

right Line which joyns the 2 points P or p, the intersections of the two Circles, will be perpendicular to F D, and bissected by it; wherefore it is evident that the points



P, *p* will form a curved Line which will pass thro' T, and the points P, *p* another, which will pass thro' I: Moreover any one will easily see, that the Lines P T P, and *p* I *p* will not include Space, neither both together, nor one alone; and that they encrease infinitely, and run off continually from the Line I T, and are continued from the Point C, one on one side, and the other on the other.

DEFINITIONS.

1. The curved Lines P T P, *p* I *p*, are each of them separately called an *Hyperbola*, and conjoyntly *Opposite Hyperbola's*.

2. The point C is called the *Center* of the *Hyperbola*, or of the *Opposite Hyperbola's*.

3. The right Line I T is called the *Determinate Axis*.

4. The right Line N C M, which passes thro' the Center C, and is perpendicular to I T, is call'd the *Indeterminate Axis*.

5. The

5. The Points F and D, the *Foci*.

6. The right Lines as P O, drawn from the Points of one *Hyperbola*, or the Oppoſite *Hyperbola's* perpendicular to one of their Axes, are call'd *Ordinates* to that Axis.

7. All the right Lines which paſs thro' the Center C are called *Diameters*, thoſe which meet the Oppoſite Hyperbolas are *determined*, and thoſe which do not are *Indetermined*.

8. A right Line which meets the Hyperbola but in one point, and does not paſs within it, is called a *Tangent* to it in that Point.

PROP. I.

The Hyperbola being formed; I ſay, that the Square of P O *an Ordinate to the determinate Axis* I T, *is to the Rectangle* I O T, *as the Rectangle* I F T, *or* T D I *which is equal to it, to the Rectangle* I C T, *which is the Square of* C T.

Draw the Line F P, and prolong it to B, ſo that P B may be equal to P D; on P as a Center, and with the Radius P D deſcribe the Circle B G D, cutting F P in A, and I T in D and G: F A will be equal to I T, by *the Generation*, which let be divided into two equal parts in R. By reaſon of the Circle A D G B, the Rectangle F A × F B = the Rectangle F D × F G. wherefore F A. F D :: F G. F B, and conſequently their halves F R or C T. C D : : C O. R P; and *by Compoſition* C T. C T + C D :: C O. C O + R P; and *alternately* C T. C O : : C T + C D. C O + R P; and farther by *Compoſition*, C T. C T + C O = I O :: C T + C D = I D. C T + C D = I D + C O + R P.; But C T + C D + C O + R P, or their equals F R, F C, C O, R P being added together are = F P + F O; wherefore C T. I O :: I D. F P + F O.

In like manner taking the proportion above, C T. C D : : C O. R P. by diviſion, C T. C D — C T :: C O. R P — C O, and alternately C T. C O : : C D — C T. R P — C O; but farther by Diviſion C T. C O — C T

— CT (= OT) :: CD — CT (= DT.) RP — CO — CD + CT; But RP — CO — CD + CT, or their equals RP — CO — FC + FR, are equal to the difference of FP and FO; wherefore CT. OT :: DT. FP — FO.

If the laſt proportions, in the two preceding Sections, be multiply'd into one another Antecedents into Antecedents, and Conſequents into Conſequents, we ſhall have CT *q*. Rectangle IOT :: Rectangle IDT. Rectangle FP + FO × FP — FO. = PO *q* (*by the Lemma. Prop.* 1. *Ellipſis*) W. W. D.

If the Circle BDA touch IT in D, *i. e.* if the Point O fall upon the point D, which is the ſame thing, the Demonſtration will be the ſame but ſtill more ſimple, for by that means more different Quantities wou'd become equal.

COROL.

It is evident, that the Squares of the Ordinates to the *Axis* are to one another as the Rectangles contain'd under the parts of that *Axis* comprehended between it's Extremity,

and the points where it meets those Ordinates.

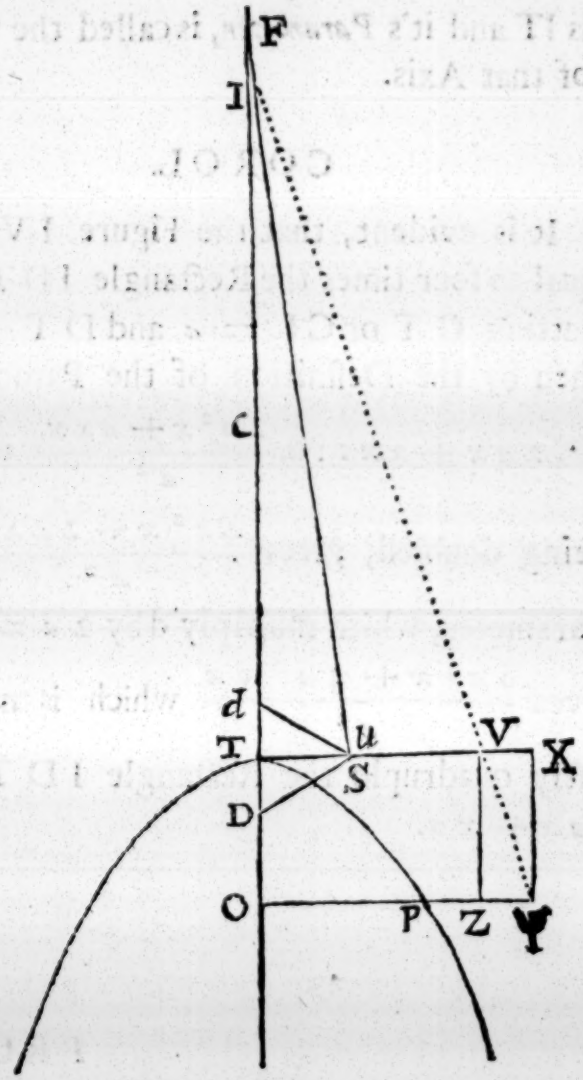
PROP. II.

The right Line P *p drawn between the oppoſite Hyperbola's, parallel to the determinate Axis* I T, *meets the indeterminate Axis* N M *in a point as* M, *which point* M *divides* P *p into two equal parts.*

This Propoſition is ſufficiently evident, *from the Conſtruction* ;(*See Fig. Pag.* 83.) for if D be made the Center of a Circle, and F P it's Radius ; F the Center of another Circle, and DP it's Radius ; the two Circles will interſect one another at the point *p*, ſo that *p o* being drawn at right Angles to the Axis I T, will be equal to P O, and likewiſe *p* M will = P M.

DEFINITION.

If you make as C T *q*. Rectangle I D T :: C T. T *u* ; the Line T V double of T *u* is called



called the *Parameter* of the determinate *Axis*. ([redacted])

And

And the Rectangle I V made by the Axis IT and it's *Parameter*, is called the Figure of that Axis.

COROL.

It is evident, that the Figure I V is equal to four times the Rectangle I D T, for putting C T or C I = a and D T = x; then by the Definition of the Parameter $a^2 . 2ax + xx :: a . \frac{2a^2x + axx}{a^2}$ which being doubled, gives $\frac{4a^2x + 2axx}{a^2}$ = Parameter, which multiply'd by $2a$ = IT, gives $\frac{8a^3x + 4a^2xx}{a^2}$ which is manifestly quadruple the Rectangle I D T = $2ax + xx$.

PROP.

PROP. III.

The Square of PO *an Ordinate to the determinate Axis,* IT *is equal to the Rectangle* OTXY, *apply'd to the Parameter* TV; *the Altitude of which Rectangle is* TO, *the part contain'd between the Extremity* T, *and the Foot of the Ordinate* O; *which Rectangle exceeds the Rectangle* TOZV *by the Rectangle* VZYX, *which is ſimilar and ſimilarly poſited to the Figure. See the preceding Fig.*

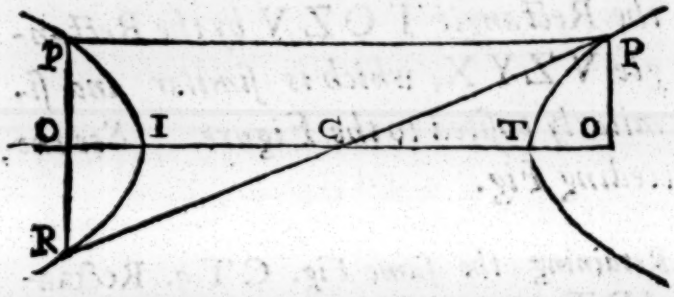
Retaining the ſame Fig. CT *q.* Rectangle IDT :: Rectangle IOT. PO *q*; but by the preceding *Cor. Prop.* 2. CT *q.* IDT :: IT *q.* ITV :: IT. TV; wherefore *ex æquo* IT. TV :: Rectangle IOT PO *q*; but IOT. YOT :: IO. YO :: IT. TV; therefore *ex æquo* Rectangle IOT. PO *q* :: Rectangle IOT. Rectangle YOT; and conſequently YOT = PO *q.* W. W D.

PROP.

PROP. IV.

If from any point of one of the oppoſite Hyperbola's, be drawn the Diameter P C, *and continued beyond* C, *it will meet the other of the oppoſite Hyperbola's, and likewiſe be biſſected in* C.

Draw P O an Ordinate to the Axis I T,



and make C *o* = CO; and draw likewiſe the Ordinate *p o* R on each ſide of the Axis; R O = P O (*by Prop.* 2.) wherefore R P will paſs thro' the Center, and will but make one right Line with P C which was firſt drawn, and will alſo be biſſected in C. W. W. D.

PROP.

PROP. V

A Perpendicular to the Axis as TX, *meeting the Hyperbola in the Vertex* T, *touches the Hyperbola in that point.*

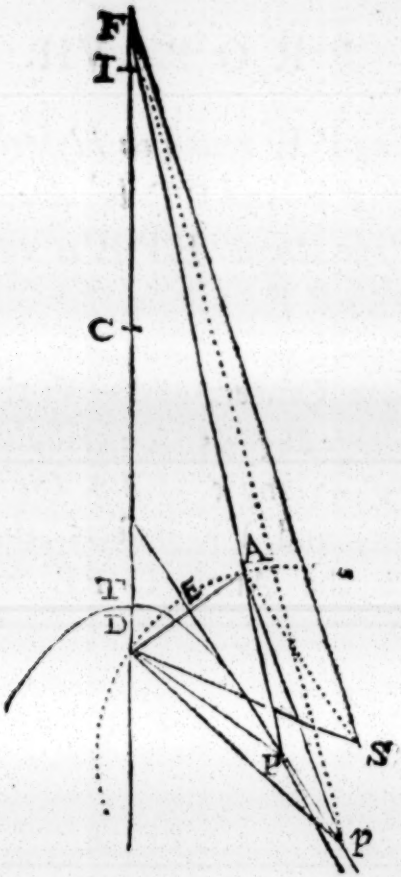
Suppoſe XT do not touch the Hyperbola, but meet it in ſome other point as S, which is different from T; (*See Fig. Pag.* 90.) take T*d* = TD, and then will F*d* be = IT, and S*d* = SD; but (*by the Generation of the Hyperbola*) IT + SD, or their equals F*d* + S*d* = FS, *i. e.* one ſide of the Triangle FS*d* is equal to the Sum of the other two ſides F*d*, *d*S, which is abſurd; therefore the perpendicular XT does not meet the Hyperbola except in the point T: I ſay alſo, that it can never poſſibly paſs within the Hyperbola, tho' it be produced; for it is evident (*from the Generation*) that the Hyperbola runs off more and more from the indeterminate Axis, which is parallel to XT; therefore XT touches the Hyperbola in T; which was to be ſhewn.

PROP.

PROP. VI.

Things being put as in the firſt Propoſition, and having drawn D A, *and biſſected it in* E; *I ſay, the right Line* P E *touches the Hyperbola at the point* P.

Suppoſe it otherwiſe, and that P E meet the Hyperbola in any other point as *p*, then (*by the Generation of the Hyperbola*) the difference of the Lines F *p*, *p* D equal to the difference of the Lines F P, P D is equal F A; but the Hypotheſis P E is perpendicular to A D, and is biſſected in E; wherefore *p* A = *p* D; therefore the difference of the ſides F *p* and *p* A is equal to the other ſide F A, *which is abſurd.* But if the right Line E P produced beyond P, fall within the Hyperbola, it is manifeſt, that the Hyperbola will paſs on the other ſide of E P, with regard to the Axis I T; therefore having drawn from any of it's points as S, the Lines S F and S D to the Foci; (*by the Generation*) the Line I T (or F A which is equal to it) + S D is = S F; but the point S being without the Line



Line E P with regard to the point D, the Line S A muſt be leſs then S D, wherefore S A + F A will be leſs than S F, which is abſurd, for 2 ſides of any Triangle are always greater than the third; *therefore the Prop. is. true.*

PROP.

PROP. VII.

If the Line P E *touch an Hyperbola in* P, *and if* P F *and* P D *be drawn to the Foci, the Angle* F P E *is equal to the Angle* D P E.

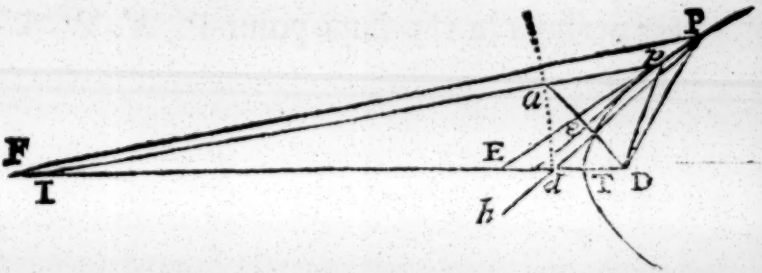
On P as a Center, and with the Radius P D (*See Fig. Pag.* 95.) deſcribe the Circle D A; the Triangle D P A (*by Prop.* 6.) is Iſoſcelar, and P E biſſects the Baſe; wherefore the Angle D P E = F P E. W. W. D.

PROP.

PROP VIII.

The ſame things being put as in Prop. 6, *I ſay, there can be but one Line as* PE *which ſhall touch the Hyperbola in the ſame point* P.

Let PH alſo touch the Hyperbola in P if it be poſſible; from the Focus D having drawn D *a* perpendicular to P *h*, and hav-



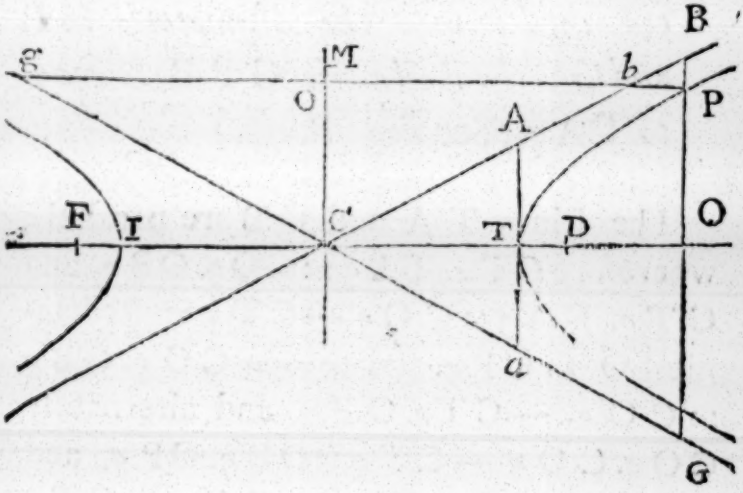
ing deſcribed the Circle *d a* from the Center F, and with the Radius F *d* = I T, it will meet D *a* in the point *a*; having biſſected D *a* in *e*, and having drawn F *a* produced until it meet the Hyperbola in *p*; and having joyn'd *p* D and *p e*, *a p* and *p* D will be equal (*by the Generation*) and *p e* drawn within the Iſoſceles Triangle *a p* D biſſecting the

Baſe D *a*, will be perpendicular to D *a*; wherefore it will be parallel to P *h*; but it touches the Hyperbola in *p* (*by Prop.* 6.); therefore *p e* will meet a Line touching the Hyperbola in the point T, in ſome point without the Hyperbola, ſeeing there are both Tangents; wherefore the parallel P *h* paſſes within the Hyperbola, for the Hyperbola goes from P to *p*, and the Ordinate to the point P, meets P *h* within the Hyperbola, which is abſurd, for P *h* was ſuppoſed a Tangent: Therefore there can be drawn but one Line as P E, which will touch the Hyperbola in the ſame point P. W. W. S.

Definition

Definition of the Aſſymptotes.

If the right Line *a* T A be drawn perpendicular to the Axis I T, at one of it's Ex-



tremities as T, and T A *q* be made equal to the Rectangle I D T; having taken T *a* = T A; thro' the Center C and the points A, *a* draw the Lines A C, *a* C indetermined both ways from the Center; theſe Lines are called *Aſſymptotes* to the Hyperbola, or the oppoſite Hyperbola's.

COROL.

It is evident that the Aſſymptotes to one Hyperbola are alſo Aſſymptotes to the other that is oppoſite.

PROP. IX.

If the Ordinate of an Hyperbola be produced until it meet the Asymptotes in B *and* G; *the Rectangle* G P B *is equal to* T A *q.* (See Fig. P. 99.)

The Lines T A and O B are parallel; wherefore C T *q*. T A *q* :: C O *q*. O B *q*, and C T *q*. T A *q* :: C O *q* — C T *q* = Rectangle I O T. O P *q*. and *ex æquo* C O *q*. O B *q* :: C O *q*. — C T *q*. O P *q*, and alternately C O *q*. C O *q* — C T *q* :: O B *q*. O P *q*, and by division C O *q*. C O *q* — C O *q* + C T *q* = C T *q* :: O B *q*. O B *q* — O P *q*, and alternately C O *q*. O B *q* :: C T *q*. O B *q* — Rectangle O P *q* = G P B: but in the first Propotision C O *q*. O B *q*: :C T *q*. T A *q*; wherefore the Rectangle G P B = T A *q*. W. W. D.

PROP. X.

If from ſome point as P *in an Hyperbola be drawn the right Line* P *g parallel to the Axis* I T, *meeting the two Aſſymptotes in* b *and* g; *I ſay, the Rectangle* g P b *is equal to* C T q.

From the poſition of the Aſſymptotes, b g is biſſected in O by the Axis C M; and *from ſimilar Triangles,* O B q. (*See Fig. in Pag.* 99.) C O q = P O q :: C O q, or o P q. o b q; and by Diviſion, O B q. O B q — O P q :: o P q. o P q — o b q; and alternately, and by inverſion, O B q — O P q = Rectangle G P B. o P q — o b q = Rectangle g P b :: O B q. o P q = C O q : but O B q. C O q :: T A q. C T q; wherefore *ex æquo* Rectangle G P B. Rectangle g P b :: T A q. C T q; but Rectangle G P B = T A q. (*by the laſt Prop.*) wherefore Rectangle g P b = C T q. W. W. D.

PROP. XI.

The Hyperbola and it's Aſſymptote approach continually to one another the farther both of 'em are produced, and yet will never meet, tho' P B, the part of the Ordinate contain'd between the Hyperbola and it's Aſſymptote, may be found ſuch that it ſhall be leſs then any Line given.

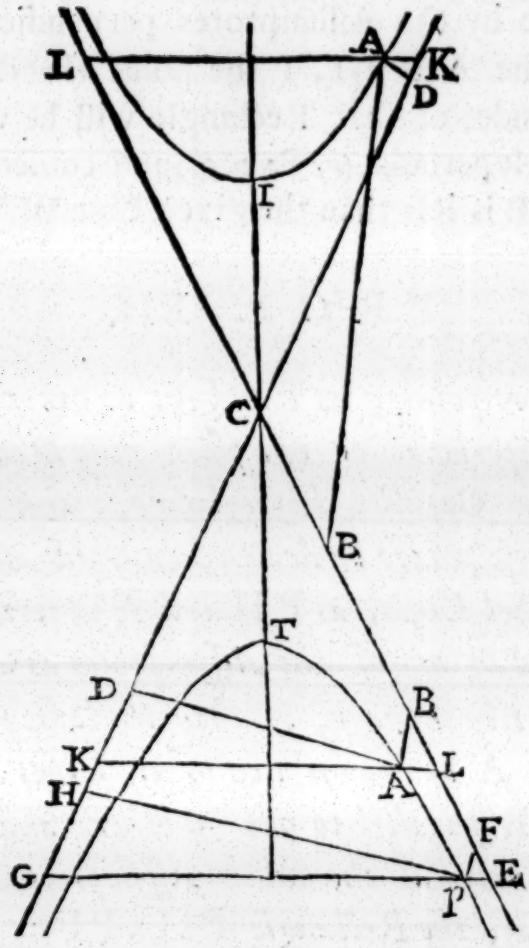
The Rectangle G P B is equal to T A *q*. (*by Prop.* 9.) (*See Fig. in Pag.* 99.); but G B encreaſes the farther it is diſtant from C; wherefore P B continually diminiſheth; but it can never become equal to nothing, for then the Rectangle G P B wou'd be equal to nothing, which wou'd contradict the forementioned Prop. wherefore the Hyperbola and it's Aſſymptote can never meet: Then if a Rectangle be made, one of whoſe ſides ſhall be leſs than the given Line, which Rectangle ſhall be equal to T A *q*; the Sum of the two ſides of this Rectangle, as G B, being apply'd within the Angle made

made by the Aſſumptotes perpendicularly to the Axis I T, P the point that divides the ſides of that Rectangle will be within the Hyperbola (*by Prop.* 9.) and conſequently P B is leſs than the given Line W.W.D.

PROP. XII.

If two points, as P *and* A, *be taken in an Hyperbola, or one point only in each of the Oppoſite Hyperbola's, and thro' theſe points be drawn two right and parallel Lines, as* P H *and* A D *terminated by one of the Aſſymptotes as* C D; *in like manner, if thro' the ſame points* P, A *be drawn two other Lines* P F, A B *parallel to one another, and terminated at the other Aſſymptote* C B; *I ſay, the Rectangle* P H × P F = A D × A B.

Through the points P, and A, having drawn the Perpendiculars to the Axis E P G, and L A K terminated by the Aſſymptotes in E G, L K; the Triangles



EPF and LAB, PGH and KAD are similar, by reason the Lines that compose them are parallel; wherefore EP. PF :: LA. AB and PG. PH :: AK. AD, wherefore multiplying these two Proportions into one another, Antecedents into Antecedents, and Consequents

quents into Conſequents; there will ariſe this Proportion EP × PG. PF × PH :: LA × AK. AB × AD; but the Rectangle EPG equals Rectangle LAK; for (*by Prop.* 9.) they are equal Squares of one and the ſame right Line wherefore the Rectangle PF × PH is equal to the Rectangle AB × AD. W. W. D.

COROL.

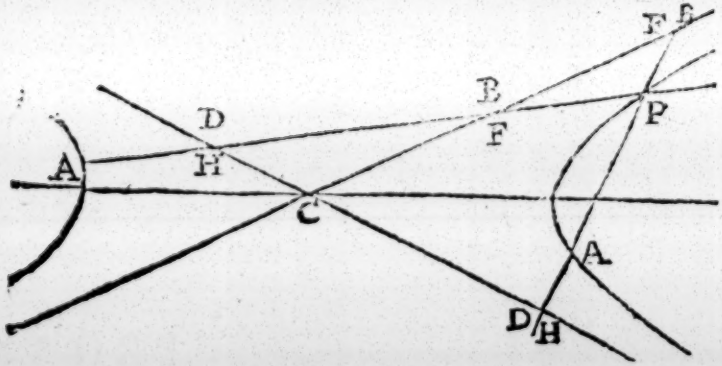
It is evident that what we have demonſtrated above for two points only, may be demonſtrated in like manner for any number of points whatſoever.

PROP.

PROP. XIII.

Draw a right Line at pleaſure as P A *meeting one Hyperbola, or the two Oppoſite Hyperbola's in the points* P *and* A, *and the Aſſymptotes in* F *and* H, *and then the parts of this Line* P F, A H, *comprehended between the Hyperbola, or the Oppoſite Hyperbola's, and their Aſſymptotes are equal.*

By the preceding Prop. the Rectangle P F



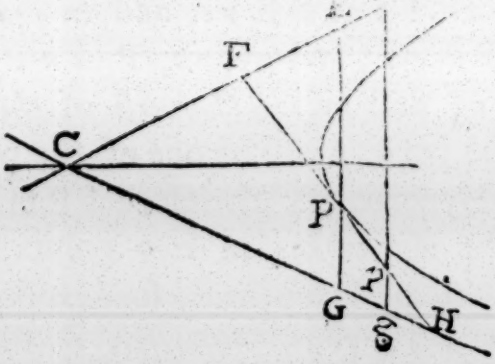
× P H is equal to the Rectangle A B × A H, for the Lines P F, A B, as alſo P H and A D are parallel, being joyn'd in a ſtreight Line:

Line: But the Sums or the Differences of H F, D B the ſides of thoſe equal Rectangles are alſo equal, being they are the ſame right Lines; wherefore the ſides of theſe Rectangles are alſo equal, *viz.* P F = A D, and P H = A B.

PROP. XIV.

If a right Line, as F H, *touch an Hyperbola in* P; *I ſay, that it meets the Aſſymptotes in* F *and* H, *and is biſſected by the point of Contact* P.

1. Thro' the point P draw E P G an Ordinate to the Axis, and if the Tangent in P do not meet the *Aſſymptotes*, it will be



parallel

parallel to one of 'em, which let be C G if it be poſſible. (*By Prop.* 11.) One may find a Line leſs then P G, which being parallel to P G, let it be comprehended between the Hyperbola and the Aſſymptote; this Line (*See Fig in the foregoing Pag.*) produced will neceſſarily meet the Tangent F P H parallel to C G within the Hyperbola, which is abſurd, for F P H is ſuppoſed to be a Tangent; therefore a Tangent meets both the Aſſymptotes, which is the firſt part of the Prop.

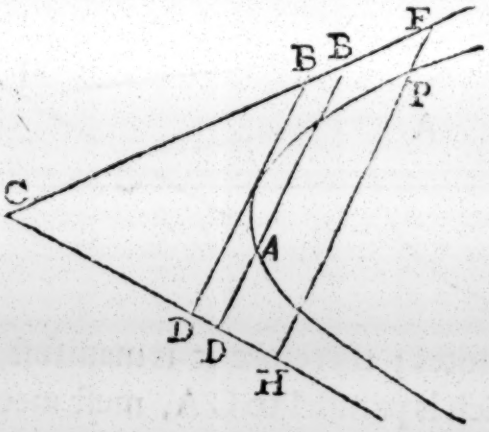
2. If P H be unequal to P F, let it be greater if poſſible; having cut off H *p* = F P, draw *e p g* parallel to E P G; then becauſe *p* F = H *p*, and *g e* parallel to G E, *p g*. P G :: H *p*. H P, and H *p*. H P, or their equals F P. F *p* :: P E. *p e*, and *ex æquo p g*. P G :: P E. *p e*; wherefore, P G × P E = *p g* × *p e*; and *by the Converſe* of *the 9th. Prop.* the point *p* will be one of the points of the Hyperbola; therefore F P H will meet the Hyperbola in two points P, *p* contrary to the Hypotheſis, for it is ſuppoſed to be a Tangent. *Wherefore the Prop. is true.*

PROP

PROP. XV.

Thro' the points P, A *of an Hyperbola, draw the right Lines* F H, B D *parallel to one another, and meeting the Aſsymptotes each of 'em in the points* F, H, B, D; *I ſay, the Rectangle* P F × P H *is equal to the Rectangle* A B × A D *or* A B*q*, *if* B D *touch the Hyperbola in the point* A.

This Propoſition is evident from *Prop.* 12. *and* 14.; for the Lines P F, P H, and



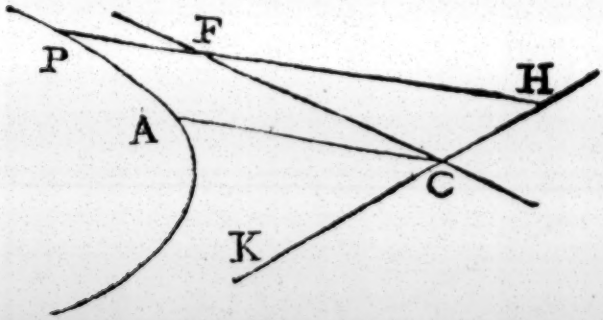
A B, A D are parallel; and if B D be a Tan-

Tangent in the point A, it is divided by the point A into two equal parts (*by Prop.* 14.)

PROP. XVI.

If from any point P *of an Hyperbola be drawn* A *right Line parallel to the determinate Diameter, as* CA; *I ſay,* PH *meets the Aſſymptotes in* F *and* H, *and the Rectangle* PF × PH = CA *q.*

For CA is within the Angle made by the



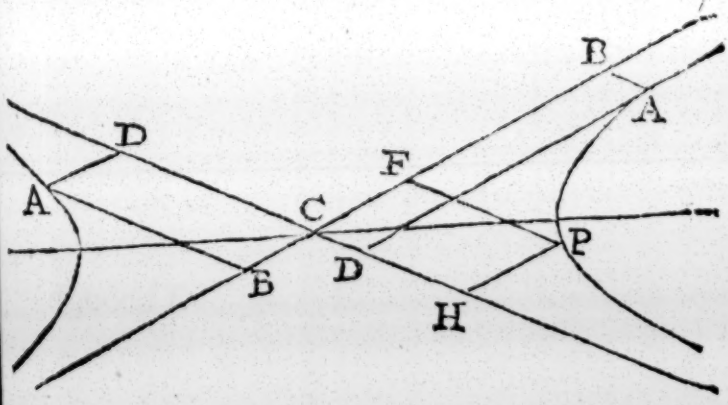
Aſſymptotes; therefore it is manifeſt, that PH which is parallel to CA, muſt meet the Aſſymptotes in F and H: But (*by Prop.* 12.) the Lines AC and PF being parallel, and meeting

meeting the Asſymptote C F; and A C and P H being alſo parallel, and meeting the other Aſſymptote C H, the Rectangle A C x A C = A C q is = Rectangle P F x P H. W. W. D.

PROP. XVII.

If from the points P, A *of an Hyperbola or Oppoſite Hyperbola's, be drawn* P F, P H, *and* A B, A D *parallel to it's Aſſymptotes; the Parallelogram* P F C H *is equal to the Parallelogram* A B C D.

By Prop. 12. the Rectangle P F x P H is



equal Rectangle A B x A D; wherefore P H.

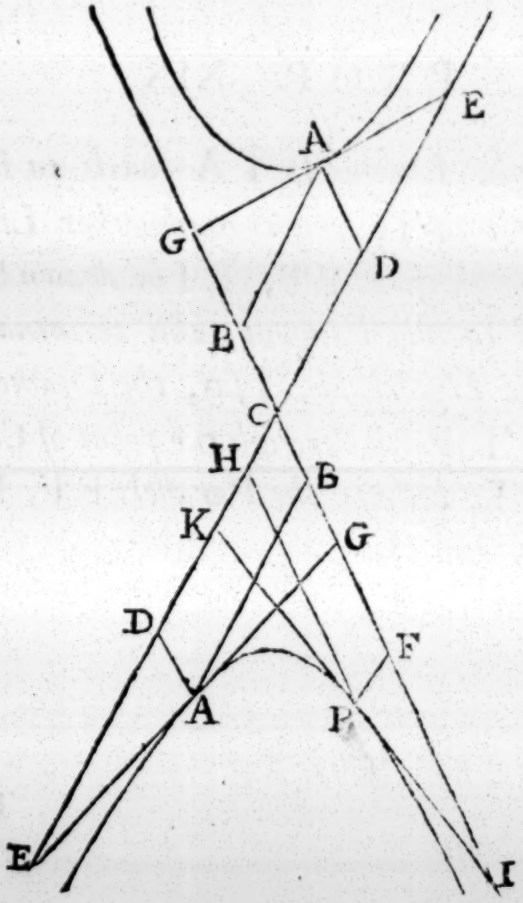
P H. A D :: A B. P F; but the Angle F P H is equal Angle B A D; wherefore the Parallelograms C P and C A are equal: W. W. D.

PROP. XVIII.

If the right Lines K I, E G *touch an Hyperbola or Oppoſite Hyperbola's; I ſay, the Triangles* K C I *and* E G C *comprehended between the Tangents and the Aſſymptotes are equal.*

Through

Thro' the points of Contact P and A, having drawn the Parallels to the Aſſymptotes A B, A D, and P F, P H (*by the prece-*



dent Prop.) the Parallelograms C A and C P are equal; but becauſe the Tangents are biſſected by the points of Contact (*by Prop.*

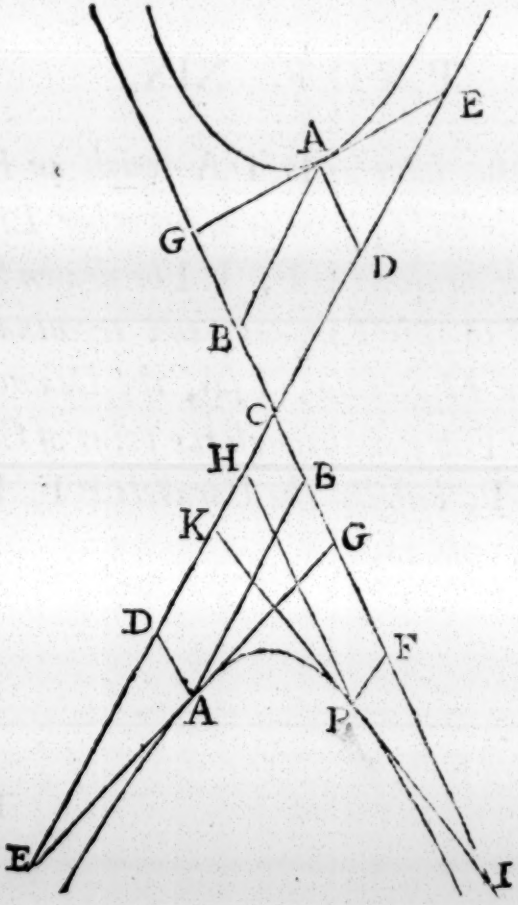
P H. A D : : A B. P F; but the Angle F P H is equal Angle B A D; wherefore the Parallelograms C P and C A are equal: W. W. D.

PROP. XVIII.

If the right Lines K I, E G *touch an Hyperbola or Oppoſite Hyperbola's; I ſay, the Triangles* K C I *and* E G C *comprehended between the Tangents and the Aſſymptotes are equal.*

Through

Thro' the points of Contact P and A, having drawn the Parallels to the Aſſymptotes A B, A D, and P F, P H (*by the precedent Prop.*) the Parallelograms C A and C P are equal; but becauſe the Tangents are biſſected by the points of Contact (*by Prop.*

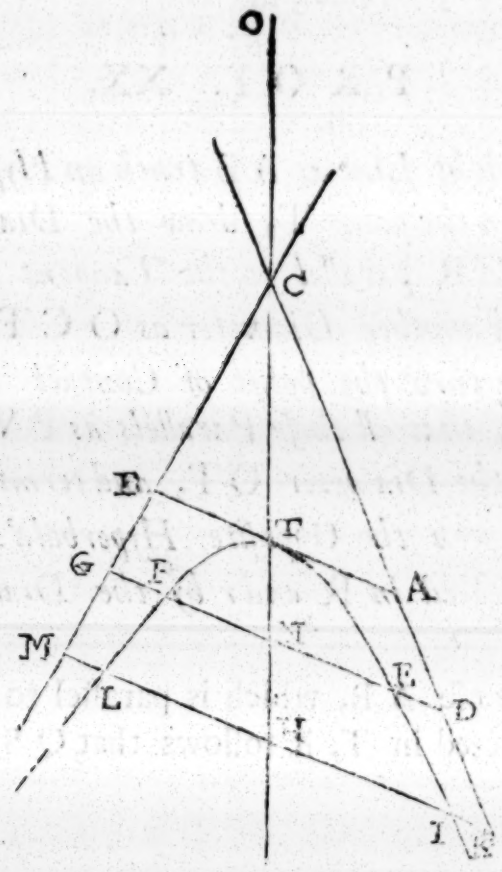


14.) the Parallelogram C P ſhall be ½ the Triangle C I K; and the parallelogram C A ſhall alſo be ½ the Triangle C E G; wherefore the Triangles are equal. W. W. D.

PROP. XIX.

If a right Line as B T A *touch an Hyperbola in* T, *and as many other Lines as you pleaſe as* E F, L I *be drawn Parallel to this Tangent, and terminated in the Hyperbola; I ſay, the Diameter* O C T *drawn through the point of Contact* T, *biſſects the Parallels* E F, L I *in* N *and* H.

The

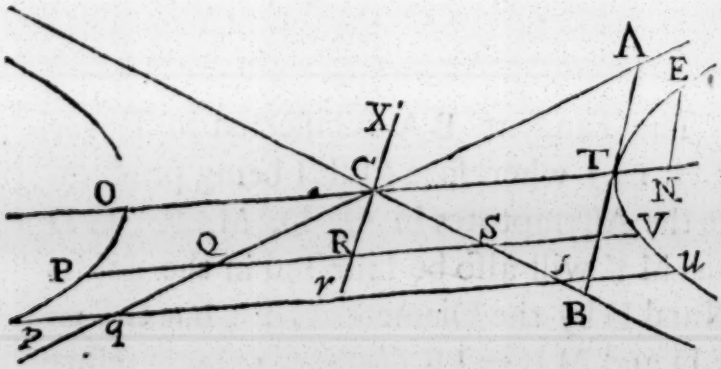


The Tangent B A is biſſected in T (*by Prop.* 14.) wherefore EF, L I being produc'd to the Aſſymptotes in G, D, M, K; G D and M K will alſo be biſſected in the points N and H by the Diameter C T; but G F = E D and M L = I K (*by Prop.* 13) wherefore F N = N E, and L H = H I. W. W. D.

PROP. XX.

If a right Line as A B *touch an Hyperbola in the point* T, *draw the Diameter* X C R *parallel to the Tangent* A B, *and another Diameter as* O C T, *passing thro' the point of Contact* T; *I say, that all these Parallels as* P V, *put to the Diameter* O T, *and terminated between the Opposite Hyperbola's are bissected in* R *and r by the Diameter* X C R.

Because A B, which is parallel to X C, is bissected in T, it follows that Q S, and



q s comprehended between the Assymptotes,

ptotes, and the Parallels to C T will likewise be bisected in R and r; but P Q = S V, and $p q = s u$; wherefore P R = R V, and $p r = r u$. W. W. D.

DEFINITIONS.

1. The Diameters O T, X R whose Properties we have demonstrated in the two preceding Propositions, are called *Conjugates* to one another. (*See Fig. Prop.* 19. *and* 20.)

2. A right Line as E N parallel to one of the Conjugates as X C is called an *Ordinate* to the other O T, and reciprocally V R is an *Ordinate* to X C.

PROP. XXI.

The Square of E N *Ordinate to a determinate Diameter as* O T, *is to the Rectangle* O N × T N, *as* A T q. *is to* C T q (See Fig. Prop. 19.)

From *similar Triangles*, D N q. T A q :: C N q. C T q; and by Division D N q — T A q = G E D *by Prop.* 15. (which is =

 E N q)

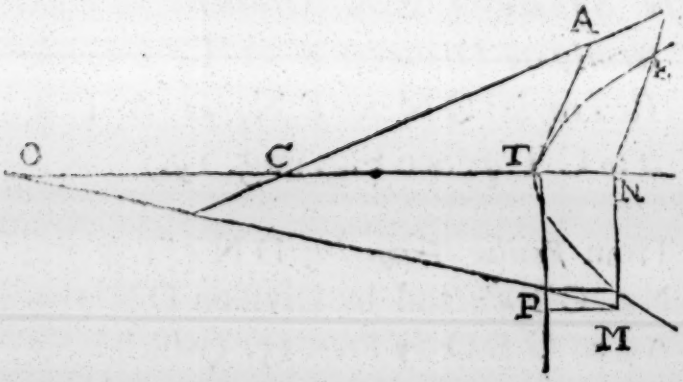
EN q). is to TA q :: CN q — CT q (equal to the Rectangle ON × TN) CT q; wherefore EN q TA q :: ON × TN. CT q. Q. E. D.

COROL.

It is evident, that the Squares of the Ordinates EN, IH to a determinate Diameter OT, are as the Rectangles ON × TN, OH × TH, made under the parts of that Diameter, which are comprehended between its Extremities O and T, and N and H the points where the Ordinates meet it.

DEFINITIONS.

1. Let CA be to TA as TA is to a



Part

part of T P; the right Line T P is called the *Parameter* of the determinate Diameter O T.

2. The Rectangle O P contained under the determinate Diameter O T and it's Parameter is called the *Figure* of that Diameter O T.

PROP. XXII.

The Square of E N *an Ordinate to a determinate Diameter as* O T, *is equal to the Rectangle* T M *apply'd to the Parameter* T P, *whose Altitude is* T N *the Abscissa, which Rectangle exceeds* P N *by the Rectangle* P M, *which is similar and similarly posited to the Figure* O P. (See Fig. preced. Defin.)

By the formation of the Parameter, O T. T P :: C T *q.* T A *q.* and (*by the precedent Prop.*) C T *q.* T A *q* :: Rectangle O N × T N. N E *q*; but the Rectangle O N × T N. M N × T N :: O N. M N, and O N. M N :: O T. T P, and *ex æquo* the Rectangle O N ×

T N. N E q : : Rectangle O N x T N. M N x T N; wherefore the Rectangle M N x T N is equal to N E q. W. W. D.

COROL.

If the Parameter T P be equal to the Diameter O T, it follows, from the formation of the Parameter, that T A will be equal to C T *i. e.* equal $\frac{1}{2}$ the ſame Parameter. But it is alſo evident (*by Prop.* 18.) that the Triangle A B G is rectangled in B, if the Diameter drawn from the Center C to the point of Contact A, be equal to the Tangent A G, comprehended between the point of Contact A, and the Aſſymptote; for the Triangle C A G will be Iſoſcelar, and the Baſe C G muſt be biſſected in B, by the Line A B, which is parallel to the other Aſſymptote C E; wherefore if the Aſſymptotes of an Hyperbola interſect one another at right Angles, all their Diameters and Parameters will be equal: And if in an Hyperbola a Diameter be equal to it's Parameter, all the others will be ſo too, and the Aſſymptotes will be at right Angles. And moreover it is farther evident,

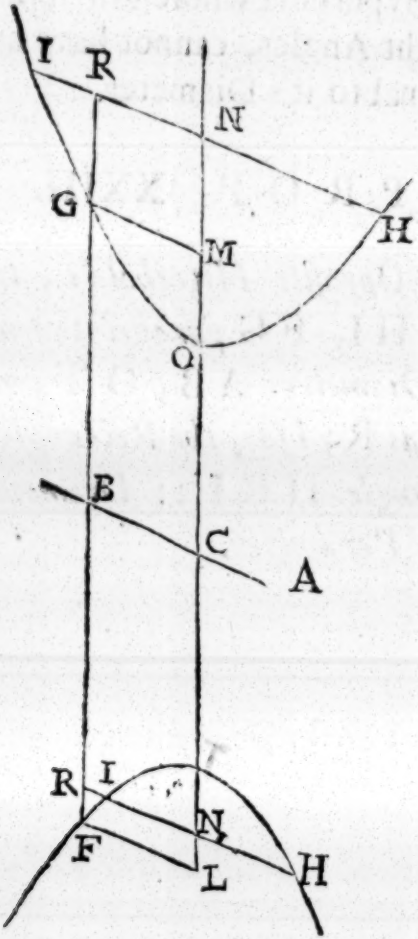
that

that an Hyperbola whoſe Aſſymptotes are not at right Angles, cannot have any Parameter equal to it's Diameter.

PROP. XXIII.

If in the Oppoſite Hyperbola's, two right Lines HI, FG *parallel to two Conjugate Diameters* AB, OT, *meet in a point as* R; *I ſay, the Rectangle* FRG. *Rectangle* HRI :: *Diameter* OT. *to it's Parameter.*

Having



Having drawn F L and G M Ordinates to the Diameter O T, they will be parallel to A B (*by the Definition of Conjugate Diameters*) and O T biſſects H I in N (*by Prop 19.*)

In

In like manner G F is biſſected by the Diameter A B; but C O = C T, and M L = G F; wherefore T L = O M (*by Prop.* 21.) L F *q*. N I *q* :: Rectangle O L T. Rectangle O N T; and by Diviſion, if the point R be without the Oppoſite Hyperbola's L F *q* — N I *q* (= Rectangle H R I.) is to N I *q* :: Rectangle O L T — Rectangle O N T (= M N L or F R G, for M O and T L are equal) O N T; therefore the Rectangle H R I. N I *q* :: Rectangle F R G. Rectangle O N T.

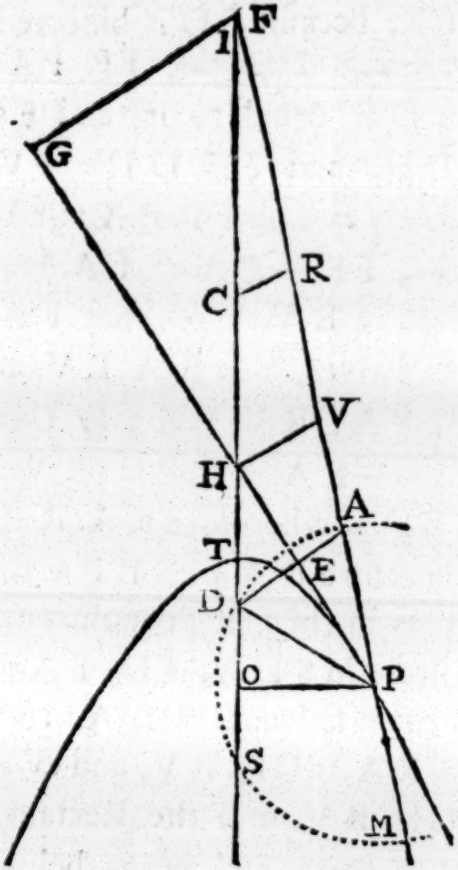
But if the point R be within one of the Oppoſite Hyperbola's, by Diviſion, N I *q* — L F *q*, or G M *q*, or N R *q* (L F, G M, and N R being all three equal) equal to the Rectangle H R I :: Rectangle O N T — Rectangle O L T (O L T being equal to the Rectangle O M T which is equal Rectangle F R G) is to the Rectangle O N T. But in theſe two caſes, (*by Prop.* 21.) and the Definition of the Parameter, Rectangle O N T. N *q* :: Diameter O T. it's Parameter; wherefore *ex æquo* the Rectangle F R G. H R I :: Diameter O T. it's Parameter. W. W. D.

PROP.

PROP. XXIV.

In an Hyperbola as P T, *whoſe Axis is* I T, *and it's Center* C, *let* H *be the point where a Tangent as* P H *meets the Axis, and let* P O *be an Ordinate to the Axis paſſing thro' the point of Contact* P; *I ſay,* CH. CT :: CT. CO

Let



Let the points F, D be the Foci, and from P as a Center; and with the Radius P D, having deſcribed the Circle M S D A, and drawn the Line D A; the Tangent P H will biſſect D A in E *(by Prop. 6.)* From the points F, C, H, having drawn F G, C R, and

and H V parallel to D A, F A will be biſſected in R, becauſe F D is biſſected in C.

From ſimilar Triangles, F P. P A : : F G. A E, or D E equal to it, and F G. D E : : F H. D H, and F H. D H : : F V. V A; conſequently *ex æquo* F P. P A : : F V. V A; *by Diviſion*, F P — P A = F A. P A : : F V — V A = 2 R V. V A; but F A. 2 R V : : as their halves, *viz.* R A R V. and *ex æquo* R A. P A : : R V. V A; *by Compoſition*, R A. R A + P A = R P : : R V. R V + V A = R A; wherefore R A. R P : : R V. R A; therefore R A q = R P × R V.

Now, as in the firſt Propoſition, becauſe of the Circle M S D A, F D. F A : : F M. F S, and their halves C D. R A : : R P. C O; But C D. R A : : C H. R V, and *ex æquo* R P C O : : C H. R V. and the Rectangles under the extreams and means being equal, C H × C O is = R P × R V; but the Rectangle R P × R V was demonſtrated above to be equal R A q, and R A = C T by the *Conſtruction*; wherefore C T q = C H × C O therefore C H. C T : : C T. C C O. Q. E. D.,

COROL

CORLLARY.

Since C H. C T : : C T. C O by inverting the Demonſtration of the 1ſt Article of this Prop. it may be prov'd, that I O. O T : : I H. H T, and the Line I O is ſaid to be divided *Harmonically* in the points I, O, T, H.

A TREATISE OF *Conick Sections.*

PART. IV.

The Deſcriptions of the Conick Sections upon a Plain.

THE Method of Deſcribing *Curve Lines* upon a Plain, by the continual motion of a Point with Machines, is ſo ſubject to errour, that it ought not to be uſed above once, leaſt its perplexity ſhou'd diſcourage the Learner; and I am apt to think there is nothing more Requiſite than

to find an Infinite number of Points, after an eaſy manner, thro' which may be drawn the Curve Line deſired; and as it frequently happens that there is occaſion but for a ſmall part of a curve line, ſo there may be found ſo great a number of Points, that there can be no conſiderable errour in drawing the *Curve* through the Points ſo found The Deſcriptions of the 3 Conick Sections which I have given at the beginning of the Demonſtrations of every one, being generally received for the moſt ſimple, when the *Foci* or *Axes* are known, it wou'd be needleſs to ſeek for any other, when thoſe things are given. But becauſe it often happens, that one may have occaſion from other *data* to deſcribe the Sections, for inſtance to deſcribe *the Parabola* having given only one of the Diameters, with its Parameter, and the Angle the Ordinates make with that Diameter. To deſcribe the Ellipſis having given 2 conjugate Diameters; and to deſcribe the Hyperbola having given the Angle made By the Aſſymptotes and any point in the Curve, or a Diameter, and its Parameter; or in one Word,

word, to deſcribe any of the 3 Sections, having given a Diameter with one of its Ordinates. I have therefore given the Deſcriptions of theſe Lines in the caſes propoſed.

PROB. I.

A Diameter of a Conick Section being given with an Ordinate to that Diameter, to find the Parameter; and in the Ellipſis to determine its conjugate Diameter.

For the Parabola.

If you ſay, as the Intercepted Diameter is to the Ordinate, ſo the Ordinate to a third Proportional. It is Evident by the Definition of the Parameter of a Diameter in the Parabola that this third Proportional is the Parameter Sought.

For the Ellipſis and Hyperbola.

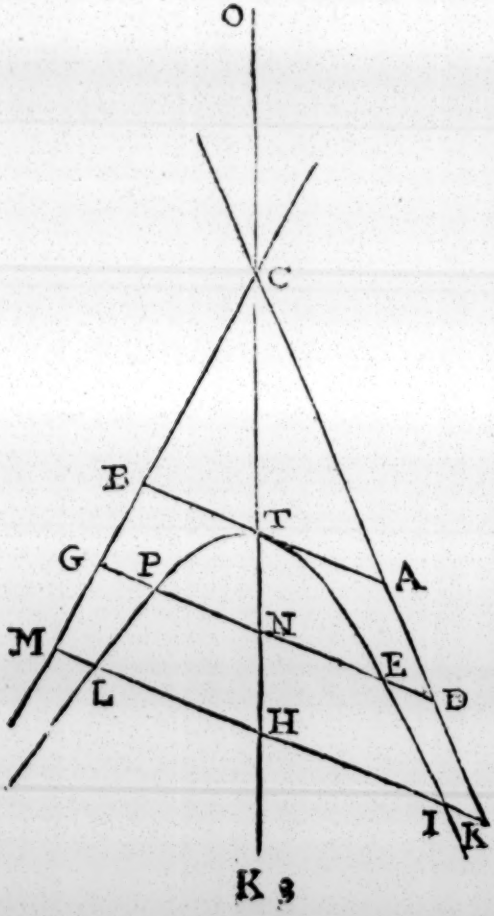
If you make as the Rectangle under the parts of that Diameter, made by the

Rencounter of the Ordinate to this Diameter, is to the ſquare of that Ordinate, ſo is the Diameter to a fourth Proportional, which fourth Proportional thus found will be the Parameter of that Diameter: And in the Ellipſis having found a mean Proportional between the Diameter, and its Parameter. And having drawn thro' the Center a line parallel to the Ordinate, and equal to the mean Proportional found and which is biſſected by the Center, we ſhall have the conjugate Diameter to the Diameter propoſed; as is evident *by the Definition of the Parameter of a Diameter in the Ellipſis and Hyperbola, and by Prop.* 17. *and* 18. *of the Ellipſis.*

PROB

PROB. II.

In an Hyperbola, a determinate Diameter with its Parameter being given, and the Angle which this Diameter makes with its Ordinates, to describe the Aſſymptotes.



Let

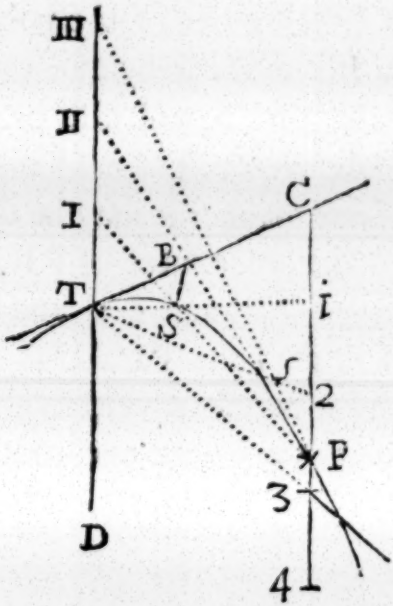
Let the Diameter given be O T, and the Angle O N E, which this diameter makes with its Ordinate, thro' the extremity T draw A T B parallel to E N; and having found A B a mean Proportional between the Diameter O T and its Parameter, biſſect it in T, and thro' the Center, and the Points A and B, produce C A and C B on each ſide the Center C. I ſay that C A, C B, are the Aſſymptotes of the propoſed Hyperbola; *as is evident by the Generation of the Parameter of an Hyperbola.*

PROB. III.

The Right Line I T D *being given for the Diameter of a Parabola, and the point* P *for one of its points, together with the line* C T *touching it at* T *the extremity of that Diameter: Or rather the angle* D T C = *to the Angle the Diameter makes with its Ordinates; To deſcribe the Parabola.*

Thro' the point P having drawn C P parallel to T D, and prolong'd it downwards from

from P, take C *i*, *i* 2, 23 *&c.* Equal to one another, of what bigneſs you pleaſe, ſo you do but begin at the point C where C P meets T C; produce T D upwards, and ſet off the ſame parts from T, making



'em thus T I, III, II, III *&c.* Having drawn P I, T *i* meeting one another in S, I ſay the point S is one of the points of the Parabola required.

Draw S B parallel to D T; and *from the ſimilar Triangles* T B S, T C *i*, and I T S, P *i* Si

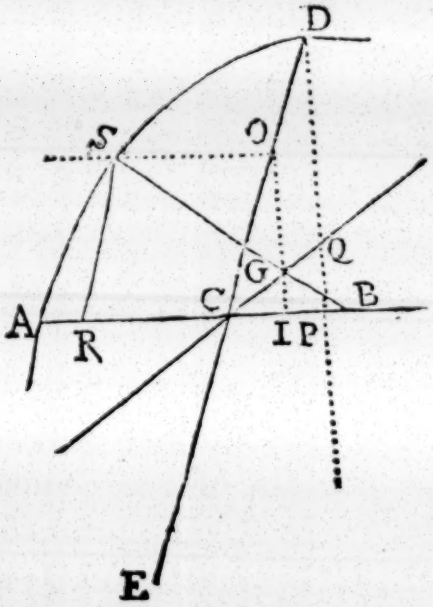
BS. C i = TI :: TB. TC; TI. iP + TI or C i = CP :: TS. T i :: TB. TC wherefore BS. CP in a duplicate ratio of TB to TC; But likewiſe TB q: TC q in a duplicate ratio of their ſides TB, TC; wherefore TB q. TC q :: BS. CP, and the point P being one of the points of the Parabola, It is evident by *Cor.* 3. *Prop* 1. *of the Parabola*, that S. will alſo be a point in the Parabola required.

In the ſame manner it may be demonſtrated that S the rencounter of P II, and T 2, and Z that of P III and T 3, and ſo on, will be points in the ſame Parabola, and if the parts C i *&c.* be taken very ſmall you may find 3 points very near one another.

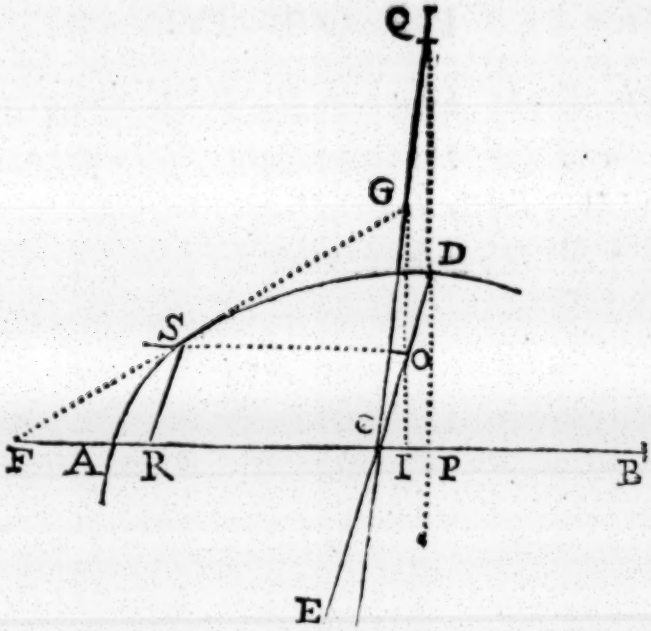
PROB.

PROB IV.

The Right Lines A B, E D, *being given for the conjugate Diameters of an Ellipsis intersecting one another in* C: *To describe that Ellipsis.*



Thro' D the extremity of any one of the Diameters, draw D P perpendicular to the other conjugate Diameter A B, and produce it both ways, and upon this line take D Q on either side of D = C A = ½ A B; thro'



thro' the center C, and the point Q, draw the line C Q produc'd both ways from C; from any point O of the Diameter D E having drawn the Right Lines O S parallel to C A, and O G parallel to O Q; and upon the Center G, and with the Radius G S = C A having deſcribed the Ark S meeting O S in S: I ſay the point S, is one of the points of the Ellipſis required.

From

From the Similar Triangles C D Q, C O G.
C D q. C O q = R S q :: D Q q = G S q = C A q. O G q, But

O G q = difference of G S q and S O q for the Triangle G O S is Rectangled in O, and after the ſame manner, G O q is = difference of the ſquares of C A and C R, which are equal to G S and S O; and the difference of C A q and C R q is = Rectangle B R A; for A B is biſſected in C, wherefore C D q. R S q :: C A q. B R A and (*by Prop.* 18.) the Point S is a point of the Ellipſis required, and ſo may an infinite number of other points be found.

Another manner of Deſcription for the ſame Conſtruction.

Having drawn C Q as above, and taking G ſome point in the Line C Q for a Center, and with the Radius G F = C P, having deſcribed the Ark F cutting C B in F, and having prolong'd F G to S, and drawn G S = C A; I ſay the point S is a point in the Ellipſis required.

Thro'

Thro' the point G having drawn I G O parallel to D Q P; F G = P Q. G I :: D Q = G S. G O; wherefore if S O be joyned, drawn it will be parallel to C A, and it may be demonſtrated as above, that the point S is a point in the Ellipſis ſought.

If the Lines A B, D E *be the Axes.*

If the Lines A B, DE be the Axes, the preceding method will ſerve; for it is evident that the Lines C Q. and C D will be joyned, and that Q P will be the difference in the 1 Fig. and the ſum in the 2 Fig. in the preceding Prob. of the 2 Semi-axes; and that the Points as G ought to be taken upon C D; and the reſt of the conſtruction and demonſtration will be the ſame as before.

Another manner of Deſcription by the help of a Circle.

In the preceeding Figure, the Reader muſt ſupply the Circle, and the Lines which are not there; which is very eaſy to do.

If

If the quadrant of a Circle be deſcribed with the Radius C A, and having joyned A D; if from any point R of the Diameter C A you erect a perpendicular and continue it to the Circumference of the Circle, this perpendicular will be a mean Proportional between B R and R A; and having drawn R O parallel to A D, and O S parallel to C A, = to the mean Proportional between B R, R A. I ſay the point S is in the Ellipſis ſought.

By the Conſtruction C D *q.* C A *q*:: Rectangle E O D. Rectangle B R A = O S *q*; wherefore C D *q*. C A Q or E D *q*. B A *q*:: Rectangle E O D. O S *q*; and *by* 18. *Prop. Ellip. the point* S is a point in the Ellipſis ſought.

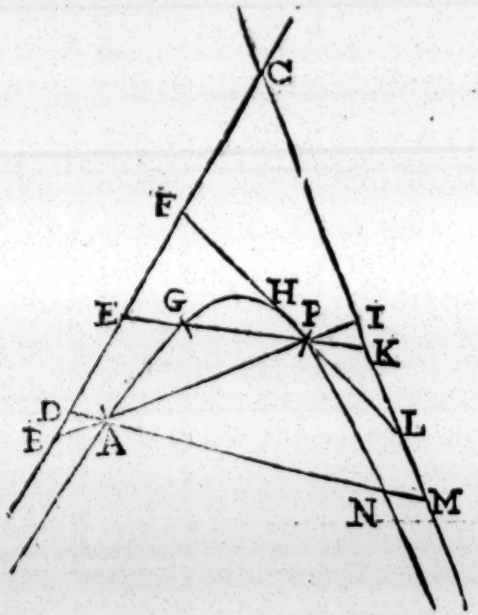
Note in this ſuppoſition O S C R, are not = as in the Figure

PROB.

PROB. V.

The Aſſymptotes CD, CM *being given, with a point in the Hyperbola; 'tis required to deſcribe the Hyperbola.*

Thro the Point P, draw BPI, EPK, EPL at pleaſure, terminated at the Aſſymptotes; and make BA = PI, EG = PK.



FH = PL, I ſay the points A, G, H, are in the Hyperbola required; this is evident *by the* 13. *of the Hyperbola.*

And

And if thro any of the points found you ſtill draw other lines as D M, make M N = D A, and it is evident for the ſame reaſon, that the point N is in the Hyperbola.

FINIS

ERRATA.

IN the Preface, Page 5. Line 12 and 13. for *concourſe*, &c. read *Vertex of the Parabola as the determined Focus.* p. 2. l. ult. f. PF, r. PP. p. 21. l. ult. r. PO *q* : E G *q*. p. 24. l. 8. after *of* r. *contact*. p. 36. place C at the Centre of the Figure. p. 39. l. 20. r. 36. p. 48. place T at the Extremity of the Tranſverſe in the Fig. p. 53. make L an E in the Fig. p. 68. place C at the Centre of the Fig. p. 73. l. ult. f. *to* L r. *to* E. p. 79. l. 6. for D I *q* r. O I *q*. lb. f. A D *q* r. A B *q*. p. 89. C muſt be placed in the middle between I and T, P muſt be placed where O Z meets the Curve, and ☤ muſt be made a Y. lb. l. 2. dele *See Fig. Pag.* 93. p. 95. in the Fig. draw the Line D A, and place E at the middle of it. p. 109. Fig. place A where B D meets the Curve on that ſide towards H. p. 123. make C a G, and P a E in the Fig. p. 127. l. 18. f. *Pag.* r. *Prop.* p. 129. l. 1. f. *a part* r. *half*. p. 133. l. 17. after H R I, r. *is to* N I *q*. p. 148. make the upper C an O, and that I next B a P in the Fig. p. 150. l. 3. f. *joyned* r. *drawn*. l. 151. l. 12 f. G A *q* r. C A *q*.